THE HEBREW CALENDAR:
A Mathematical Introduction

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PROGRAM I

USING THE TIME UNITS OF THE HEBREW CALENDAR

INTRODUCTION

Why should YOU study the Hebrew calendar?

One of the major identifying signs of the Church of God is the observance of the Sacred Festivals. As you study the twenty-third chapter of Leviticus, you will notice that God employed a calendar to indicate when each holy day must be properly kept during the year. The Jews were given the responsibility of preserving that calendar for the rest of the world.

Since it is the responsibility of the Church to announce the time of each festival to the congregations, detailed understanding of the Hebrew calendar is not even necessary for a lay church member. When a holy day is to be kept is not for the individual Christian to decide.

On the other hand, the education you are privileged to receive as an Ambassador College student equips you with a special depth of biblical understanding. Shallow or sketchy knowledge of basic background areas would lessen one's effectiveness. But working out the calendar principles yourself is going to widen your perspective.

You already know that the holy days portray God's master plan of salvation for mankind. Shouldn't you also have a working knowledge of the very calendar which houses God's Sacred Festivals?

This is why a study of the Hebrew calendar is included in Theological Research I-II.

THE PURPOSE OF THESE LEARNING PROGRAMS

Your study of the Hebrew calendar in this course has two major facets. One is the historical development of the calendar. This is the primary function of the class lectures. The other is for you to achieve needed computational facility with the calendar itself.

Fortunately, the Hebrew calendar requires surprisingly little mathematical sophistication. A fifth grade background in arithmetic will suffice! Nevertheless, a certain number of skills and concepts must be learned for you to become adept at working with the Hebrew calendar. These programs are designed to provide you with that understanding and practice.

Just what will you be able to accomplish when you complete this series of learning programs?

For any year, such as 4 BC, 31 AD, 1520 AD, and 1979 AD, you will correctly determine the dates on a common Roman calendar of the holy days listed in Leviticus 23.

How long will this operation require? With nothing but a pencil and a blank sheet of paper, you might need anywhere from thirty to forty-five minutes. If you use a table of reduced numbers (which is included in one of the programs), it might take you only ten to fifteen minutes.

That skill is the OVERALL GOAL of this series of programs. Another less tangible aim is to give you the confidence that you can actually SUCCEED in working a calendar problem!

To that end, each learning program takes a necessary part of the main goal, and gives you the practice needed to become adept at it. Success will breed success as you progress!

The first page of each learning program has a clear statement of what you must be doing by the time you complete the program. You might think of each program as a "checkpoint" on route to your destination. Be sure you can accomplish each program goal before going on to the next one.

One word of caution. Have you ever learned mathematics simply by glancing over the textbook or watching someone else work through a problem? No, you can't! The exercises in each program are entirely for your benefit. In most cases, these exercises will be worked out in detail later in the program. This is for you to have a model with which to compare your own procedures and to check your work immediately for errors.

But WORK you must! Proverbs 4:13 says to "take fast hold of instruction; let her not go." Learning the mathematical operations of the Hebrew calendar will take ACTIVE EFFORT. With this diligence, you will achieve the exhilaration only success can bring.

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PERFORMANCE GOAL 1A:

Without hesitation or uncertainty you will write (or recite) from memory in any order the following time relationships:

1 part (chalyek)	=	76 moments (regaim)
1 hour	=	1080 parts (chalakim)
1 day	=	24 hours
1 week	=	7 days
1 lunar month	=	29 days 12 hours 793 parts
1 common year	=	12 lunar months
1 intercalary year	=	13 lunar months
1 nineteen year cycle	=	235 lunar months, or 12 common years & 7 intercalary years

PERFORMANCE GOAL 1B:

With only paper and pencil (or pen), you will accurately add, subtract, multiply, and divide the time relationships listed in lA as necessary to compute specified problems. Large numbers will be "reduced to lowest terms" when requested. For example,

28 hours reduces to 1 day, 4 hours
31 hours 650 parts reduces to 1 day, 7 hours, 650 parts.

PERFORMANCE GOAL 1A

In order for you to become adept with calendar calculations, you must become very familiar with a minimal number of time relationships. Some of these you already know; others require memorization. These numbers will occur so frequently that you simply must know them. Otherwise, the lessons will take much longer to understand and you will feel frustrated in the process. Learn them now -- for your own benefit.

For a detailed yet concise description of the time elements of the Hebrew calendar, consult either of the following references:

"The Jewish Encyclopedia", volume 3, "Calendar" (either new or old edition).

Burnaby, Sherrard Beaumont, "Elements of the Jewish and Muhammadan Calendars". London: George Bell & Sons, 1901. 554 pages. (See chapter II, page 21.)

Here is a brief explanation of some of these time elements to assist your memorization.

DAY: Genesis 1:5 shows that the day begins in the evening. "And the evening and the morning were the first day. " Although each day begins at sunset, 6 PM is the arbitrary commencement of a new day for CALENDAR CALCULATIONS.

Christ put his divine approval upon dividing the day into twenty-four hours. See John 11:9, which states, "Are there not twelve hours in the day?" Context shows that this verse refers to the daylight portion of a twenty-four hour period. For any given day the periods of darkness and light are usually unequal. The total length of a day, however, is always 24 hours--except for Divine intervention!

HOUR: Instead of being divided into minutes and seconds, the hour is divided into parts, or chalakim. One hour consists of 1080 parts, or 3600 seconds. Using parts instead of minutes and seconds has the advantage of eliminating fractions. (The smaller unit, the moment or rega, is seldom needed for fundamental calculations.) Both the hour and the part are considered fixed units anywhere on earth, just as the minute and the second are non-varying time elements.

MONTH: A lunar month is the time needed for the moon to revolve around the earth. Even though this period varies from month to month, 29 days 12 hours 793 parts is the traditional average used for calculation. Actual calendars cannot be based upon 29 1/2 days, so the Hebrew calendar incorporates months of 29 and 30 days.

YEAR: The Hebrew calendar has two basic types of years, common and intercalary. (The latter term is also called "embolismic.") An intercalary Hebrew year will have 30 additional days, so it can also be called a leap year. By contrast, recall that the Roman leap year has 366 days instead of 365. You should also remember that the following terms are interchangeable for the Hebrew calendar:

LEAP (year or month) = embolismic (year or month) = INTERCALARY(year or month)

Common years will have 353, 354, or 355 days. Leap years may have 383, 384, or 385 days.

19 YEAR CYCLE: The Western world is accustomed to a solar year of 365 1/4 days, since the Roman calendar in common use is solar. This means that a given month of the year will always occur during the same season. January, for example, is invariably a winter month.

On the other hand, the Hebrew year by itself does not closely match the length of a solar year. Twelve months which are each approximately 29 1/2 days results in a year which has only 354 days -- about eleven days less than a solar year of 365 1/4 days. A common Hebrew year is thus SHORTER than a Roman year.

What about a Hebrew leap year? Thirteen months of 29 1/2 days is 383 1/2 days, which is LONGER than a solar year.

God has ordained that the holy days must be kept "in their seasons" (Lev. 23:4). He also appointed BOTH the sun and the moon "for signs, and for seasons, and for days, and years" (Genesis 1:14). This means that the calendar which God designed to house His sacred festivals would be luni-solar. The months must occur at the proper times for the holy days to fall within the proper season of the year.

How, then, are the Hebrew lunar months related to the solar year? Every 19 solar years (of 365 1/4 days), the moon revolves around the earth 235 times, each "lunation" being on the average 29 days, 12 hours, 793 parts. This remarkable astronomical relationship makes it possible to combine common years and leap years together within a fundamental pattern that repeats itself every nineteen years:

```
12 common years (12 months each)is 144 months   (each month is
 7 leap years (13 months each)  is  91 months     29d, 12 h,
----------------------------------------------    793p)
19 Hebrew years is 235 months  = 19 solar years
```

You should be aware, however, that 235 lunar months is about an hour and a half less than 19 Julian years. (To be precise, 235 lunations is 1 hour, 485 parts LESS than 19 Julian years.)

The 19-year cycle is also known as the cycle of Meton, or the Metonic cycle.

Be sure you have MEMORIZED the time relationships listed at the beginning of this section! Don't assume that you know them "pretty well."

PERFORMANCE GOAL 1B

The source of many errors in calendar calculations is faulty arithmetic. Most occur in the basic addition, subtraction, and multiplication operations. All of these skills you learned before high school. Consequently, some review and practice are absolutely necessary. WORK the problems; don't be deluded into thinking that glancing over them will suffice.

ADDITION: The key to successful computation rests in one simple rule:

add only likes together

Just as 3 apples and 4 oranges don't equal 7 apples, neither does adding 11 hours to 4 days equal 15 hours. Never put a number down without being positively certain that you know what it represents. The best way is to label every number: 671 p is 671 parts. Another way is to keep like quantities in clearly defined columns:

```
                     days      hours      parts
                    ------    -------    -------
                      1         12         103
                      3         15        1186

This is the same as:  1 d       12 h       103 p
                      3 d       15 h      1186 p
```

Select a format you like and use it consistently. Here are some sample problems:

```
5 d       15 h        175 p
```

```
    + 1 d       7 h        801 p
    ------------------------------
      6 d      22 h        976 p
```

So long as you add only like quantities, no outstanding difficulties will happen.

```
      6 d     237 h       1189 p
    +24 d     107 h       2159 p
    ------------------------------
     30 d     344 h       3348 p
```

The numbers can become very large, as you can see. Later on you will learn how to reduce these to lowest terms.

```
      8 d      13 h        198 p
     18 d      23 h        432 p
     13 d      45 h       1835 p
  +   7 d      96 h        103 p
   -------------------------------
     46 d     177 h       2568 p
```

Adding a whole string of numbers together will often occur. Don't be afraid to double and triple-check your arithmetic!

Solve the following problems below. If you aren't "comfortable" by the time you complete them, try to make up a few of your own for additional practice.

```
1.1)         6 d          17 h            879 p
     +       3 d          14 h            198 p
     --------------------------------------------

1.2)        67 d          95 h            777 p
     +     275 d         777 h           2589 p
     --------------------------------------------


1.3)        55 d         178 h           976 p
            31 d           1 h           134 p
           178 d          23 h          1937 p
           225 d           9 h         11581 p
           308 d         768 h           649 p
     +      29 d          12 h           793 p
     ----------------------------------------------------------------


1.4)       131 d          29 h           433 p
           227 d           8 h           191 p
             0 d           1 h           485 p
     +     130 d          12 h           883 p
     ----------------------------------------------------------------
```

The answers to these problems appears below:

1.1)	9 d	31 h	1077 p
1.2)	342 d	872 h	3366 p
1.3)	826 d	991 h	16070 p
1.4)	488 d	50 h	1992 p

REDUCTION: Before attempting to subtract or multiply these numbers, it will be helpful for you to understand how they are reduced.

You have learned how to reduce quantities such as inches to feet and yards, or seconds to hours and minutes, back in junior high mathematics:

45	inches reduces to	1	yard	0	feet	9	inches
49	inches reduces to	1	yard	1	foot	1	inch
436	seconds " "	0	hours	7	min.	16	seconds
3600	seconds " "	1	hour	0	min.	0	seconds

Reduction of large numbers is not difficult. An EQUIVALENT way of saying exactly the same thing is all that you are doing. Both quantities are equal.

The primary units in your calculations will be days, hours, and parts. Each of these quantities are related to each other, as you have already learned from performance goal 1A.

1. 5) Complete the following:

________ parts = 1 hour

________ hours = 1 day

The answers, of course, are 1080 parts in an hour, and 24 hours in a day. This means that 1080 parts can be written as:

0 days 1 hour 0 parts

In like manner, 24 hours can be shifted to the days column:

1 day 0 hours 0 parts.

Take a quantity like 3 d, 49 h, 1400 p. How do you reduce it? What you want to do is transfer all EXCESS WHOLE HOURS contained in the parts column to the hours column. Then you will take out all the WHOLE days contained in the hours column and move them to the days column. Just like reading Hebrew, you work from right to left!

To reduce numbers, apply the following operations:

1) Divide the parts in the parts column by 1080. The whole number in the quotient represents hours.

2) Add the whole number in the quotient to the hours column. This becomes the "revised" hours. Place the remainder in the "reduced" parts column. If remainder is 0, put this number in that column.

3) Take the revised number of hours and divide this by 24. The whole number in the quotient is the number of whole days.

4) Add the whole number of days to the days column, and place the remainder in the "reduced" hours column.

```
-----------------------------------------------------------------------
Reduced numbers for days, hours, parts will have:
          any number of days 23 or less hours 1079 or less parts
-----------------------------------------------------------------------
```

Here's how to apply these rules to reduce 3 d, 49 h, 1400 p:

```
1)             1 h                        1 h  320 p
           ---------
     1080 / 1400 p
             1080
           ---------
              320


2)            1 h  320 p
     +  3 d  49 h
     --------------------
        3 d  50 h  320 p


3)          2                             2 d  2 h
         --------
     24 / 50 h
          48
         --------
           2


4)       2 d  2 h  320 p
     +   3 d
     --------------------
         5 d  2 h  320 p                  REDUCED
```

Problems with large numbers are handled the same way. The answer to 1. 2 is 342 d 872 h 3366 p.

```
1)             3 h                        3 h  126 p
           ------------
     1080 / 3366 p
             3240
           ---------
              126


2)              3 h  126 p
     +  342 d  872 h
     ----------------------
        342 d  875 h  126 p
```

```
3)          36 d                          36 d  11 h
          ---------
      24 /  875 h
             72
          ---------
            155
            144
          ---------
             11


4)      36 d  11 h  126 p
     + 342 d
     ----------------------
       378 d  11 h  126 p
```

Reducing numbers isn't difficult. Your accuracy will be enhanced if you consistently stick to a single format. Slopping numbers down haphazardly on the page is inviting computational errors.

1.6) Reduce 5, 796 parts.
1.7) Reduce 579, 600 parts.
1.8) Reduce 85 d 91 h 150,000 p.
1.9) Reduce 567 d 5228 h 254,404 p.

The answers to 1.6 - 1.9 are on the next pages. Work these problems on separate paper, then compare your calculations with the complete arithmetical details supplied.

```
1.6            5 h                         0 d  5 h  396 p answer
           ---------
     1080 / 5796 p
            5400
           ---------
             396


1.7            536 h                   536 h  720 p;  536 hours
          ---------------                             must be reduced.
     1080 / 579600 p
            5400
           ---------
             3960
             3240
            --------
              7200
              6480              22 d     22 d  8 h  720 p answer
             -------        --------
               720      24 / 536 h
                              48
                            -------
                               56
                               48
                             ------
                                8
```

```
1.8       85 d 91 h 150,000 p

     (1)        138 h                      138 h  960 p;  add this to
           ------------                                   85 d 91 h.
     1080 / 150,000 p
             108 0
           -----------
              42 00
              32 40
              ---------
               9600
               8640
              ---------
                960


     (2)        138 h  960 p
          85 d   91 h
          -------------------
          85 d  229 h  960 p


     (3)         9 d                       9 d  13 h;  add together the
             --------                                  reduced 960 p,
          24 / 229 h                                   9 d  13 h, and
                216                                    85 d.
             --------
                 13 h


     (4)        13 h  960 p
          85 d
          ------------------
          94 d  13 h  960 p             answer


1.9       567 d  5228 h  254,404 p


     (1)            235 h                  235 h  604 p
                -----------
          1080 / 254404 p
                  2160
                ---------
                   3840
                   3240
                ---------
                    6004
                    5400
                ---------
                     604 p


     (2)          235 h  604 p
          567 d  5228 h
```

```
          --------------------
          567 d  5463 h  604 p


     (3)        227 d                     227 d  15 h
              --------
          24 / 5463 h
               48
              --------
                66
                48
              --------
                183
                168
              --------
                 15 h


     (4)  227 d  15 h  604 p
          567 d
          -------------------
          794 d  15 h  604 p
```

MULTIPLICATION: After adding days, hours, and parts, you will find that multiplication is quite easy. The important key to correct multiplication is that ALL TERMS must be multiplied! Forgetting to multiply days while you complete the operation for the hours and parts can easily happen if you aren't wary.

Look at a sample problem:

```
     2 d  16 h  595 p
                 x 3
     ----------------
     6 d  48 h 1785 p    Notice that the multiplier operates
                         on the days, the hours, and the parts.
```

Multiplication can help you solve many problems that would take much longer by straight addition. Here is an example:

How many days, hours, parts are in an AVERAGE common year?

You should easily remember:

> The number of lunar months in a common year;
> The number of days hours and parts in a lunar month.

If you cannot recall these numbers, return immediately to the first page of this section and drill intensely on the time relations listed in 1A.

A common year has 12 months. Each month has 29 d, 12 h, 793 p.
Therefore an average common year has:

```
       29 d   12 h   793 p
            x 12 months
```

```
      --------------------
       58      24    1586
       29      12     793
      -----  -----  ------
      348 d  144 h  9516 p
```

If this number were to be used in a series of computations, it could stand "as is." On the other hand, when it is the final answer, can't you see the need to reduce such a number? You may want to verify that an average common year has 354 d 8 h 876 p. (This is worth remembering.)

1.10 How many days, hours, and parts are in an average intercalary year?

1.11 What number of d, h, p are in the 19 year cycle?

1.12 7 x (5 d 21 h 589 p) = _____________

1.13 (4 d 8 h 876 p) x 12 = _____________

1.14 (18 d 15 h 589 p) x 6 = _____________

Problems 1.10 - 1.14 are worked out for you below.

```
1.10   29 d   12 h     793 p
          x 13 months
       ----------------------
       87      36     2,379
       29      12       793
       ----------------------
      377 d  156 h  10,309 p    This reduces to 383 d  21 h  589 p.
```

1.11 You may calculate this problem by two methods:
a) add the lengths of 12 common years and 7 leap years together, using the example and 3.10.
b) Find the length of 235 lunations.

Here's how method b) is solved:

```
         29 d    12 h    793 p
                      x 235 months
       ---------------------
        145      60     3965
        87      36     2379
       58      24     1586
      -----    -----  ------
       6815 d  2820 h 186355 p             This reduces to
                                             6939 d  16 h  595 p.
```

```
1.12    5 d   21 h   589 p
                    x  7
```

```
      ----------------------
        35 d  147 h  4123 p                This reduces to
                                              41 d  6 h  883 p.


1.13    4 d    8 h   876 p
                    x 12
      ----------------------
         8     16    1752
        4      8     876
      ----------------------
        48 d   96 h 10512 p                This reduces to
                                             52 d  9 h  792 p.

1.14   18 d  15 h   589 p
                   x  6
      ----------------------------------
       108 d  90 h  3534 p                 This reduces to
                                             111 d  21 h  294 p.
```

SUBTRACTION: This is the most error-prone operation for many students. Two facets of subtraction are particularly troublesome:

a) borrowing
b) negative numbers

First look at a straight-forward subtraction problem:

```
         5 d   21 h  589 p
      -  (4 d   19 h   98 p)
      -----------------------
         1 d    2 h  491 p
```

Notice that the minus sign affects all the quantities in the second line. Obviously, adding one of the units and subtracting the others isn't kosher! The first principle to keep in mind:

SUBTRACT ALL TERMS called for.

Practice on two of these problems, comparing your answer with that given.

```
1.15     6 d  6 h  883 p
      -  (1 d  5 h  785 p
      ---------------------
                                           answer: 5 d  1 h  98 p


1.16 -  (  74 d  14 h   45 p)
          111 d  21 h  294 p
      -------------------------            answer: 37 d  7 h  249 p
                                           (You CAN subtract "upside
                                             down.")
```

In the next example, "borrowing" will be necessary:

```
          8 d  23 h  403 p
     -   (6 d  22 h  528 p)
     -----------------------
```

Since 1080 parts are in an hour, 8 d 23 h 403 p can be changed to 8 d (23-1) h (403 + 1080) p, or 8 d 22 h 1483 p. Be sure you understand that neither expression is different. Borrowing merely places the number in a more convenient form for subtraction.

By borrowing quantities as needed, the problem becomes ordinary subtraction:

```
          8 d  22 h  1483 p
     -   (6 d  22 h   528 p)
     ------------------------
          2 d   0 h   955 p
```

In order to firm up the concept of borrowing in your mind, examine another illustration:

```
         3 d   3 h     0 p
     -  (1 d  18 h  6000 p)
     -----------------------
```

On this problem, you cannot simply take one hour and transfer 1080 parts. A quick estimate will tell you that at least 6 hours need to be changed to parts. Only 3 are in the hours column. Where do you get them?

First, make up for the lack of hours in the second column by converting a day (or more!) into hours. 3 d 3 h 0 p transfers to

```
     2 d  (24 + 3) h  0 parts, or  2 d  27 h  0 p.
```

Next borrow 6 hours and change them to parts:

```
     2 d  (27-6) h  (6480 + 0) p or,
     2 d      21 h        6480 p
```

Finally, subtract as required by the original problem:

```
         2 d  21 h  6480 p
     -  (1 d  18 h  6000 p)
     -----------------------
         1 d   3 h   480 p
```

Once in a while, you will find that in borrowing parts, you loose too many hours for subtracting the hours. (Instead of 18 hours, suppose you needed to subtract at least 22 hours in the above example.) When that happens, simply borrow again from the next column to the left.

Borrowing is a reverse of reduction. Remember that both quantities are equivalent -- before reduction and after reduction, or before and after borrowing. A "golden rule" for these operations:

```
     ----------------------------------------------------------------
        WHAT YOU ADD TO ONE COLUMN MUST BE SUBTRACTED FROM ANOTHER.
     ----------------------------------------------------------------
```

```
1.17    40 d  6 h   104 p
     - (35 d  2 h  1141 p)
     ----------------------
                                           answer:  5 d  3 h  43 p


1.18     136 d  123 h  5291 p
     - (  89 d   29 h  7679 p)
     --------------------------
                                           answer:  47 d  91 h  852 p


1.19     21 d  30 h   811 p
     -  (17 d  69 h  2050 p)
     ------------------------
                                           answer:  2 d  7 h  921 p
```

The last complication to hurdle with subtraction is negative numbers. By borrowing, you were able to keep numbers positive. But sometimes the arithmetic is simplified by allowing numbers to become negative-without borrowing.

There is nothing mysterious about negative time. All that it means is time BEFORE a prescribed reference point. Such a reference point is the blast-off of a moon launch. Events before lift-off are considered negative times, as you heard on many telecasts: "t MINUS eight minutes and holding." After the rocket lifts off, time becomes positive -- with reference to the ZERO lift-off.

An expression like days, hours, parts refers to a particular instant in time. When you are computing a problem, some of these numbers will occasionally become negative. Nonetheless, the expression is still identifying a precise moment. For example look at:

```
     5 d  -2 h  -810 p
```

Does this specified time occur on the fifth day or on the fourth? The answer is the fourth day. This is immediately clear if you change the expression to all positive signs by "borrowing. " These are the steps:

```
           5  d        -2  h          -810  p
        (5-1) d  (-2 + 24) h          -810  p
           4  d        22  h          -810  p
           4  d   (22 - 1) h  (-810 + 1080) p
           4  d        21  h           270  p
```

So 5 days -2 hours -810 parts is the same as 4 days 21 hours 207 parts.

You should become familiar enough with these negative expressions that you accurately complete computations involving them. In ordinary subtraction, all numbers were positive. Now you will find that subtraction and addition of numbers of either sign frequently occurring when you eventually calculate the days, hours, parts, of the month for a conjunction of the moon.

The next example illustrates how this type of problem is worked:

```
  36 d   22 h   1284 p
 -17 d +  3 h -  541 p
------------------------
  19 d   25 h    743 p    or    20 d  1 h  743 p
```

When a number is not preceded by a sign, it is taken to be positive. A few rules apply for these problems:

adding two negatives together (e.g. -5 + -3) results in a negative number larger than either (-8).

adding a negative number and a positive number together is the same as subtracting. The sign of the larger value (called the absolute value in mathematics) is the sign of the final number.

```
(-30  +  50  =  +20)
(-48  +  38  =  -10)
```

When you subtract a negative number from another negative number, you change the sign of the number you are subtracting, and then add as above. (-5 - -3 = -5 + +3 = -2)

Adding two negative numbers together results in a larger negative number. Subtracting two negative numbers results in a smaller negative number -- closer to zero. Any time you are working with negative numbers, be sure that you carry the signs with you. Although you need not specifically mark positive numbers, this may be advisable for you initially in order to keep the signs clear in your mind. Whenever you are subtracting, remember that the subtraction sign affects every number (term) in the expression:

```
      45 d  23 h   150 p
  - ( -8 d  15 h  -125 p)
  ------------------------
```

Notice how the numbers subtracted change signs:

```
45 d   23 h     150 p
+8 d  -15 h  +  125 p
----------------------
53 d    8 h     275 p
```

Try the following problem:

```
1.20     - 18 d  - 2 h  -  780 p
         -  0 d  - 1 h  -  485 p
         - 97 d  - 2 h  -  756 p
         + 74 d + 14 h  +  196 p           The answer appears below.
         + 13 d
```

```
    + 36 d + 22 h  + 1284 p
     -----------------------
```

1.20 The problem becomes easier if you total the negative quantities separate from the positive, and then combine.

```
  - 18 d  -  2 h  - 780 p             74 d    14 h   196 p
     0 d  -  1 h  - 485 p             13 d
  - 97 d  - 22 h  - 756 p             36 d    22 h 1284 p
  ------------------------      -------------------------------
  -115 d  - 25 h - 2021 p           123 d     36 h 1480 p

   123 d    36 h   1480 p
  -115 d  - 25 h - 2021 p
  ------------------------
     8 d    11 h -  541 p         This reduces (by taking one
                                    hour and changing it to
                                    parts: + 1080 - 541) to
                                    8 d  10 h  539 p.
```

1.21 Work the next problem without reducing until the final step:

```
     210 d  - 15 h  -  971 p
  - ( 37 d    19 h  - 1185 p )
    --------------------------
                                  Change signs and subtract.
                                    Your result should be
                                    173 d  -34 h  +214 p.
                                    This reduces to
                                    171 d  14 h  214 p.
```

1.22 Add 92 d 581 h 471 p to eight times (-10 d -21 h -204 p). Then subtract -152 d - 589 h 188 p from the sum. Reduce at the final answer. How many days over a full number of weeks is this?

```
First multiply: - 10 d  -  21 h  -  204 p
                                   x 8
               --------------------------
                 - 80 d  - 168 h  - 1632 p
Add:               92 d    581 h     471 p
               ---------------------------
                 + 12 d  + 413 h  - 1161 p
```

Subtracting -152 d -589 h 188 p, you change the signs and add:

```
                   12 d  +  413 h  - 1161 p
               + 152 d  +  589 h  -  188 p
               ----------------------------
                 164 d  + 1002 h  - 1349 p
```

Borrow 2 hours, and then divide 1000 hours by 24 to convert the hours column to a number less than 24. This gives:

```
               205 d  16 h  811 p
```

To find how many days this is over a full number of weeks, you divide by 7 the REDUCED number of days. 205 / 7 leaves a remainder of 2. Two days plus 16 hours and 811 p is the time over a full number of weeks.

*   *   *

After working the problems in this section, you should feel confident about your ability to successfully add, subtract, multiply, and reduce days, hours, and parts. Some may still require additional drill on these operations. Design your own problems for more practice. Have another student check your work, or see your instructor for assistance.

----------###----------

PROGRAM II

CALCULATING THE DAY OF WEEK FOR MOLAD TISHRI

PERFORMANCE GOAL 2:

Given a required Roman year, you will correctly calculate the day of the week, hours, and parts for the molad Tishri of that Roman year. This will be accomplished WITHOUT CHARTS AND TABLES.

Why do you need to be concerned with the molad of Tishri? The answer is that you must know when it occurs before you can determine the date of the Festival of Trumpets. And all other holy days within a Roman year (January-December) are ultimately referenced to that holy day. The molad of Tishri is prerequisite to most calculations involving the Hebrew calendar. Correct calculation of the molad of Tishri is thus essential for determining what dates on the Roman calendar we commonly use for God's Sacred Festivals to occur.

Certain definitions and concepts need to become crystallized in your mind:

What is a molad?
When is Tishri?
What is a bench mark?
How is time reckoned in the calculations?
What is meant by the "advancement" of the molad?

Let's find the answers to these questions.

What is the molad Tishri?

TISHRI is the seventh month of the sacred calendar. The computed time for the conjunction of the sun, moon, and the earth is called a MOLAD, from the Hebrew MOLED (plural, MOLEDOTH). This word means renewal, or rejuvenescence.

Molad of Tishri is the computed time of the new moon of the month of Tishri, which corresponds to September/October. As Tishri is also the first month of the civil Hebrew year, the molad Tishri is also the calculated astronomical commencement of the year.

Another term which you will be using is bench mark. All this means is a point of reference from which measurements can be made. Any known molad (expressed as day of the month, day of the week, hours, and parts, e.g., October 6, Sunday, 23 h, 204 p in 3761 BC) can serve as a bench mark. The most practical choice for a bench mark, however, will be the molad Tishri of year one in a 19 year cycle. 3761 BC is such a year.

In order to avoid a mental mix-up later, let's clarify how we reckon time. Just what does an expression like 4 d 7 h 503 p mean? Such an expression can be taken either of two ways: a) as an interval of time b) a time in the week.

Consider case a). Here, when you are speaking of a length of time, you assume that you are starting from a reference point of 0 days, 0 hours, 0 parts. This is just the same as clocking a track runner on a stop watch that starts ticking at the sound of the gun. A certain time span is involved, figuring from a bench mark of 0 days, 0 hours, 0 parts.

With case b), you are still reckoning time, but from a DIFFERENT reference point. In the calendar calculations you are working in Theological Research, the week begins at Sunday midnight. Sunday is regarded as the first day of the week; midnight the zeroth hour, zero parts:

The week begins: Sunday: 1 d 0 h 0 p.

An expression like 1 day 13 hours 0 parts MUST BY DEFINITION OF THE STARTING POINT refer to Sunday, the 13th hour after midnight, or Sunday 1 PM.

Likewise, 1 day 13 hours 179 parts refers to a time slightly later than 1 PM on Sunday.

The reason why we arbitrarily begin Sunday as 1 day 0 hours 0 parts is so that there will be an exact coincidence with the days of the week: 1 d is Sunday; 2 d is Monday; 4 d is Wednesday; 7 d is Sabbath. So long as all the numbers are reduced, you merely look at the number of days, and these correspond to the day of the week. (For the same reason of SIMPLICITY, we begin the day with midnight instead of 6 PM so far as the CALCULATIONS are concerned. Since the bench mark is expressed by this same reckoning, the final results will be exactly the same.)

By contrast, if we decided to define the beginning of the week, Sunday, as the zero day, and Saturday sunset (say 6 PM) as the start of Sunday, we would have the expression 0 d 0 h 0 p = 6 PM Saturday evening. Then 1 d 13 h 179 p would have an entirely different meaning, if it referred to a time in the week. 1 d would then be Monday; 13 h 179 p would be slightly after 7 AM. Do you see how much more complicated your work would then become?

Here is another distinction that can help you understand the difference in meaning between an interval of time, and a time in the week:

a) an interval of time:

parts are added to the hours are added to the days

b) a time in the week:

parts are added to the hours WITHIN the day of the week.

If 4 days 7 hours 503 parts refers to an interval of time, then 503 parts plus 7 hours is added to 4 days. Therefore the interval of time goes as far as the 7th hour(and 503 parts) of the fifth day.

If 4 days 7 hours 503 parts is to be taken as a point in the week, then 503 parts plus 7 hours is WITHIN the 4th day of the week.

Remember this difference in what is meant comes about by the way the starting point is DEFINED, and for no other reason.

During your calculation of the advancement of the molad over a full number of weeks, you may come up with a number like 0 days 4 hours and 71 parts. Be comforted by the fact that the expression is simply an interval of time by the way Sunday midnight is defined.

How can you relate a time interval to an expression of time during a week?

YOU MUST ADD AN INTERVAL OF TIME to the BENCH MARK before you can determine a real day of the week. The bench mark will implicitly tell you where the week begins. The bench mark for the year 3761 BC is Sunday, the 23rd hour 204 parts. (We could have called that bench mark Monday if we started the day at 6PM instead of midnight. But then we would be confronted with a more complicated interpretation of what the time expressions mean.)

Now transform each of the following expressions to a) a time interval b) a time during the week. Use this format:

a: _______ days, plus _______ hours _______ parts of the _______ day

b: _______ (day of the week), between _______ & _______ AM or PM

```
2.1  4 d   3 h  191 p
2.2  1 d  16 h  304 p
2.3  6 d   7 h    8 p
2.4  7 d  22 h    5 p
2.5  2 d  13 h  871 p
```

The answers are given below.

2.1 a) 4 days plus 3 hours 191 parts of the 5th day
b) Wednesday, between 3 & 4 AM

2.2 a) 1 day plus 16 hours 304 parts of the 2nd day
b) Sunday, between 4 & 5 PM

2.3 a) 6 days plus 7 hours 8 parts of the 7th day
b) Friday, between 7 & 8 AM

2.4 a) 7 days plus 22 hours 5 parts of the 8th day
b) Sabbath, between 10 & 11 PM

2.5 a) 2 days plus 13 hours 871 parts of the 3rd day
b) Monday, between 1 & 2 PM

One particularly important time interval will occur throughout your experience with the Hebrew calendar. In order to calculate the molad Tishri, you will be working with two molads:

* a known molad, such as 3761 BC -- a bench mark
* the molad of the Roman year which you are determining

The time interval between these two molads is the ELAPSED TIME. Since you are dealing with the molad of Tishri in both the required year and the bench mark, the elapsed time will be a whole number of years, such as 4520 years, 1503 years, 38 years.

Let's investigate another feature of the Hebrew calendar which we can call MOLAD "ADVANCEMENT." Now you know that the MOLAD itself doesn't really move, since the term is defined as the calculated conjunction of the sun, moon, and the earth. We are using a figure of speech very similar to sun "rise."

Here's an illustration of what is meant by the advancement of the molad. In 3761 BC, the molad Tishri was on Sunday 1 d 23 h 204 p. In 1980 AD the molad Tishri will be Tuesday 3 d 23 h 206 p. Although thousands of years have elapsed over the time span, the APPARENT "advancement" in the week of the second molad is only 2 days 0 hours and 2 parts.

Why does the molad occur on different days of the week? The length of an average lunar month is 29 d 12 h 793 p. How much greater than four full weeks is this?

```
   29 d  12 h  793 p
 -(28 d   0 h    0 p)
-----------------------------
    1 d  12 h  793 p;  about a day and a half.
```

The molads of two successive months cannot occur on the same day of the week because of this EXCESS over 28 days. In one month, if the molad were Monday 8 AM (2 d 8 h 0 p), the molad of the next month would be:

```
    2 d   8 h     0p
  +(1 d  12 h  793 p)
-----------------------------
    3 d  20 h  793 p,  or Tuesday, between 8 and 9 PM.
```

Although the second molad occurred 29 d 12 h 793 p after the first one, the second molad was "displaced" WITH REFERENCE TO THE WEEK by 1 d 12 h 793 p. Merely as a convenient label, we will refer to that apparent shift as MOLAD ADVANCEMENT. But the molad doesn't move;

only the time of its occurrence IN THE WEEK apparently advances.

The TOTAL MOLAD ADVANCEMENT is simply the EXCESS over the number of full weeks in the elapsed time from the bench mark to the molad Tishri of the desired Roman year.

Just as monthly molads will occur on different days, the molad of Tishri will advance in the week over the previous molad of Tishri. If in three years the total advance were 13 days, the Molad would be 13 days later. But in terms of the day of the week, this would be 13-7, or 6 days later in the week.

Now you will learn how to calculate the day of the week for the molad Tishri.

You will find it easier to understand how to determine the day of the week for a given molad if the steps are explained first without the mathematical details:

In order to determine the day of the week for the molad of Tishri, you must find the TOTAL ADVANCEMENT of the molad that occurs within the time span involved from a known molad, or bench mark.

What will make up that time span? From that bench mark, a certain number of years will elapse to the particular year in question:

```
molad of Tishri                                 molad of Tishri
  (bench mark)                                  of required year
                       (elapsed time)
      *________________________________________________*
```

Later on in this program, you will learn how to express elapsed time as:

```
       a whole multiple of 19 year cycles,
plus   the excess number of common years,
plus   the excess number of leap years.
```

Logically, the total advancement of the molad, or excess over full weeks in the elapsed time, for the year in question can be found by adding:

```
     the advancement due to whole multiples of 19 year cycles,
 +   the advancement due to the number of common years,
 +   the advancement due to the number of leap years.
```

An example will clarify this. If the elapsed time is 156 years, there are 8, 19 year cycles, 4 common years, and 2 leap years (you'll see how this is done later). The total advancement of the molad will be:

```
     8 times the advancement of one 19 year cycle
  +  4 times the advancement of one common year,
  +  2 times the advancement of one leap year.
```

The final step is simply adding the total excess over full weeks to whatever bench mark you started with. This figure will give you the day of the week of the molad Tishri in question.

Be sure that you understand the qualitative elements connected with the advancement of the molad. A molad "advances" with respect to a known molad because of the excess time in one average lunar month over a full number of weeks. All you are doing is adding the total advancement that occurs within the time interval from the bench mark to the molad Tishri of the year in question. You are actually finding the excess over the full weeks from the bench mark to the molad of the required year.

Your success at determining the day of the week of a required molad impinges upon:

a) Correctly finding the ELAPSED TIME from a bench mark to the required year.

b) Expressing the elapsed time in terms of multiples of 19 year cycles
+ common years in the remainder
+ leap years in the remainder

c) Calculating the molad "advancement" attributable to each element of b), then adding these together, reducing as necessary.

d) Adding the reduced advance of the molad c) to the bench mark.

ELAPSED TIME, 2A

Three possibilities exist for the elapsed time from the bench mark to the required year:

a1) Both years are AD dates.
a2) Both years are BC dates.
a3) One year is BC and the other is AD

(Although you might work backwards in time, this program will only consider problems in which the bench mark is the EARLIER of the two years.)

a1: Bench mark and required year both AD dates. The elapsed time is simply the difference between the two years. If the bench mark is 1845 AD and the required year is 1931, the elapsed time is:

$$1931 - 1845 = 86 \text{ years}$$

If the bench mark is 895 AD and the required year is 1751, the elapsed time is:

$$1751 - 895 = 856 \text{ years}$$

a2: Both years are BC dates. Many calculations use 3761 BC as a bench mark. To be mathematically consistent, it is helpful to place a negative sign (-) before all BC years. Since you will still be subtracting in order to find the difference, the second number will become positive - (-) = +. As you are primarily concerned with years after 3761 BC, or -3761, the year in question will always be more positive (closer to zero). As a check when both years are BC, expect the elapsed time to be SMALLER than 3761 years. (Use only the "absolute value" for the elapsed time, which means you can disregard the final negative sign.)

Using 3761 BC as a bench mark, what is the elapsed time to 585 BC?

$$-3761 - (-585) = -3761 + 585 = -3176$$

The elapsed time is 3176 years.

What is the elapsed time to 1486 BC?

$$-3761 - (-1486) = -3761 + 1486 = -2275$$

The elapsed time is 2275 years.

a3: Bench mark is BC and the required year is AD. 3761 BC is frequently used as bench mark for AD years. Only one thing is really different here. There is no year allotted for 0 AD or 0 BC. Mathematically, the number system does have a 0. What do you do?

Graphically, the two systems look like:

Calendar:	5 BC	4 BC	3 BC	2 BC	1 BC	1 AD	2 AD	3 AD
Number Scale:	-5	-4	-3	-2	-1	0	1	2

The number scale has one more place, a zero, than the calendar. Therefore in time intervals that cross AD/BC, you must SUBTRACT ONE from the answer you compute arithmetically. (The time span from 4 BC to 2 AD is only five years.) BE SURE YOU REMEMBER TO SUBTRACT ONE FROM THE MATHEMATICAL COMPUTATION! But only when you cross over from BC to AD.

What is the time elapsed from 3761 BC to 1000 AD?

$$-3761 - +1000 = -3761 -1000 = -4761$$

The elapsed time, however, is 4761 - 1 year, or 4760 years.

What is the time span (elapsed time) to 1974 AD from 3761 BC?

$$-3761 \text{ BC} - (+1974) = -3761 - 1974 = -5735$$

The elapsed time is 5735 - 1, or 5734 years. As a check, you should be aware that in going from BC to AD, the elapsed time will be a larger number (taken as an absolute value) than either the bench mark or the required year.

Drill yourself on elapsed time calculations with the following problems. Remember that if you are wrong here, all the rest of your calculations w ill be off!

Bench mark Required year Elapsed time

2a.1	1845 AD	1971 AD
2a.2	1883 AD	1945 AD
2a.3	3761 BC	1442 BC
2a.4	3761 BC	4 BC
2a.5	3761 BC	721 BC
2a.6	1861 BC	604 BC
2a.7	3761 BC	31 AD
2a.8	3761 BC	1859 AD
2a.9	3761 BC	1982 AD

See problems 2b.1 to 2b.9 for the elapsed time.

EXPRESSING ELAPSED TIME, 2B

Once you've determined the number of years between the bench mark and the required year, you need to express that time in terms of the Hebrew calendar.

On a practical basis, 19 year cycles are convenient. Find the number of 19 year cycles in the elapsed time, divide the elapsed time by 19. The QUOTIENT is the number of 19 year cycles.

Usually, however, this division will result in a remainder, a number from 1 to 18. This remainder will tell you the number of years elapsed in the next cycle.

As you already know each 19 year cycle consists of 12 common years and 7 leap years.

==

SINCE 142 AD (see footnote 1), the years in a cycle that are leap years are: 3 6 8 11 14 17 19

==

Be sure to memorize these numbers! (Of course, the years 1, 2, 4, 5, 7, 9, 10, 12, 13, 15, 16, and 18 are common.)

===

Before 142 AD, the leap years were: 2 5 7 10 13 16 18

===

For any remainder you acquire after dividing the elapsed time by 19, you:

1) Decide whether the required year is before 142 AD, or 142 AD and after.
2) Count the number of leap years that fit in the remainder (or you could count the common years).
3) The number of common years will be the remainder minus the number of leap years.

For example, the elapsed time from 1845 AD to 1975 AD is 130 years. This is 130/19 time cycles, or six 19 year time cycles plus 16 years remainder.

The leap years for 1975 (after 142 AD) are 3, 6, 8, 11, 14. Five leap years altogether. Since 1975 has 16 elapsed years and there are five leap years thus far, there must be 16 - 5 or 11 common years.

(footnote 1) There is some evidence that an adjustment to the Hebrew calendar may have taken place during the patriarchate of Simon III (140-163). See Cyrus Adler, "Calendar, History of," in "The Jewish Encyclopedia" (New York: Funk and Wagnalls, 1907), Vol. 3, p. 500.

Here is another example. If the bench mark is 3761 BC and the required year is 27 AD, the elapsed time is:

-3761 - (+27) = -3788 years; the elapsed time is 3787 years, since you go from BC to AD.

3787 / 19 = 199 19 year cycles, plus 6 elapsed years.

For 27 AD (before 142 AD), the leap years of the cycle are 2 and 5. With two leap years, there must be 6 - 2, or 4 common years.

Practice expressing elapsed time in terms of 19 year cycles, the number of leap years, and the number of common years. See problems 2a.1 to 2a.9.

	Elapsed time	19 year cycles	# of leap years	# common years
2b.1	126 yrs			
2b.2	62			
2b.3	2319			
2b.4	3757			
2b.5	3040			
2b.6	1257			
2b.7	3791			
2b.8	5619			
2b.9	5742			

Check answers against those below.

2b.1	6 cycles	4 leap	8 common
2b.2	3 "	1 "	4 "
2b.3	122	0	1
2b.4	197	5	9
2b.5	160	0	0
2b.6	66	1	2
2b.7	199	4	6
2b.8	295	5	9
2b.9	302	1	3

"ADVANCEMENT" OF THE MOLAD. 2c

From your previous program (1A & 1B), you are equipped to find out the required information regarding the advancement of the molad. Since a lunar month has 29 days, 12 hours 793 parts, it is 1 day, 12 hours, and 793 parts in excess of a full number of weeks. As you saw before, a monthly molad (two successive months) advances 1 d 12 h 793 p.

Knowing this, you can easily determine how much a molad "advances" in a common year of 12 months, in a leap year of 13 months, or in a 19 year cycle of 235 months.

How much does the time of a molad "advance" in the week during a common year?

```
     1 d   12 h  793 p   (the monthly "advancement",
                           or excess over 4 weeks)
                x 12         (number of lunar months in a common year)
     -----------------
     2     24   1586
     1     12    793
     -----------------                  6 days            8 hours
    12 d  144h  9516 p               -------         ---------
                                 24 / 152 h      1080 / 9516 p
                                      144              8640
                                     -------           -------
    18 d    8 h  876 p   reduced          8 h              876 parts
```

After 12 months, the molad "advancement" is 18 d 8 h 876 p. Since full weeks will not affect the days of the week, you can divide the reduced number of days by 7. So with respect to the week the molad "advancement" for a common year is:

```
          4 d  8 h  876 p.
```

You should notice an alternative way of arriving at the same number. How much does a common year exceed the number of full weeks in the year? Multiply the length of an average lunar month by 12 months:

```
     12  x  (29 d  12 h  793 p)  =  348 d  144 h  9516 p
```

Reduce this number: 354 d 8 h 876 p

Divide the reduced number of days by 7 to eliminate full weeks:

```
          4 d  8 h  876 p.
```

2c.1 Verify that an average leap year exceeds a full number of weeks by 5 d 21 h 589 p.

2c.2 Verify that a 19 year cycle exceeds a full number of weeks by 2 d 16 h 595 p.

From here on, you already are competent to handle the details of multiplying the advancement of the molad, and then adding. For 4 BC (as in 2a.4 and 2b.4) you will do the following:

```
     197  x (2 d  16 h  595 p)  molad advancement of 19 year cycles
       5  x (5 d  21 h  589 p)  molad advancement of leap years
       9  x (4 d   8 h  876 p)  molad advancement of common years
```

Multiplying and adding you will discover that this is:

```
          567 d  5228 h  254,404 p.
```

When reduced, and divided by seven (only the full reduced days are divided by 7), the advancement of the molad over a week is:

```
          3 d  15 h  604 p.
```

ADDING REDUCED ADVANCEMENT OF MOLAD TO THE BENCH MARK, 2d

The bench mark for 3761 BC is Sunday 23 h 204 p. This is reckoning from midnight. As Sunday is the first day of the week, the bench mark can be expressed as:

```
          1 d  23 h  204 p.
```

From here on, you merely add the reduced advancement of the molad to the bench mark. For 4 BC, this is:

```
              3 d  15 h  604 p
           +  1 d  23 h  204 p
          ----------------------------
              4 d  38 h  808 p
     or       5 d  14 h  808 p
```

The fifth day of the week is Thursday, so the molad occurred on Thursday, the 14th hour (2 PM), 808 parts.

So long as the final d, h, p, are reduced, you need only be concerned (for now) with the first column.

2d.1 Verify that the molad for 721 BC is 5 d 7 h 364 p.

2d.2 Verify that the molad for 31 AD is 5 d 23 h 941 p.

2d.3 At this point, you should test yourself to be sure you can fulfill the goal of program 2. Calculate the DAY OF THE WEEK of the molad Tishri for 1996 AD without consulting this program. You'll need to have memorized (or else work out again) the molad "advancement" for a 19 year cycle, a common year, and a leap year, as well as remember a bench mark.

Compare your calculations to the answer below, which is worked out in detail.

Calculate the DAY OF THE WEEK of the molad Tishri for 1996 AD.

```
     -3761
     -1996
     -----------
      5757 yrs
        -1                302 cycles
     -----------         ----------
      5756 yrs      19 / 5756
                         57
```

```
                         --------
                          56
                          38
                          ------
                          18 elapsed yrs; 6 leap yrs; 12 common yrs


     2 d  16 h      595 p              5 d  21h    589 p
           x        302  (19 yr                 x    6   (leap
     -------------------- cycles)     ------------------  yrs)
     4 d  32 h     1190               30 d 126 h  3534 p
    60   480      17850
   -----------------------
   604 d 4832 h  179690 p                 4 d   8 h  876 p
                                                    x 12    (common
                                         ------------------  years)
                                         48 d  96 h  10,512p


        604 d       4832 h         179690 p
     +   30 d        126 h           3534 p
     +   48 d         96 h          10512 p
     ---------------------------------------
        682 d       5054 h         193736 p     total advancement
     ---------------------------------------
     +  218 d       +179 h


    128 weeks        218 d                 179 h
    -------         --------            -----------
7 / 900 d      24 / 5233 h         1080 / 193736 p          (reducing)
    xxx             xxx                  xxx
    ------          --------            ----------
      4 d              1 h                 416 p     reduced advance-
                                                     ment of molad

   + (1 d            23 h                  204 p)    bench mark
-----------------------------------------------------
      5 d            24 h                  620 p
      6 d             0 h                  620 p
=====================================================
```

The day of the week of molad Tishri in 1996 AD is Friday.

----------###----------

PROGRAM III

CALCULATING THE DAY OF THE MONTH OF THE MOLAD TISHRI

PERFORMANCE GOAL 3

Without tables, you will correctly determine the day of the month of the molad Tishri for the Roman years specified in this program.

The procedure used to find the day of the month of the molad Tishri parallels that of the day of the week calculation you already learned in Program 2. Part of the work needed for the day of the month calculation is ALREADY accomplished in the day of the week calculation!

Let's review part of the day of the week calculation to see just what operations are in common. You must have a required year and the bench mark (assumed to be 3761 BC unless otherwise specified). From these two years, you can determine elapsed time between them (read over 2a again if this is hazy). Be sure you remember to subtract 1 from the total whenever you go from BC to AD. This is a common mistake!

Next, you express the elapsed time in terms of 19 year cycles, number leap years, and the number of common years (2b):

* Divide the elapsed time by 19.

* The remainder is the number of elapsed years in the 19 year cycle of the required year. If the required year is 142 AD or after, the intercalary years are 3, 6, 8, 11, 14, 17 and 19. (Before 142 AD, just subtract 1 from each of these numbers to obtain the intercalary years of that cycle.)

* Count up the number of leap years that have occurred within the cycle, including the last number. For example, if the remainder is 11, there are 4 leap years in the cycle.

* Subtract the number of leap years from the remainder, and you will have the number of common years in that cycle.

You have already done this much of the calculation in order to determine the day of the week for the molad of Tishri. And you will use this SAME information to find the day of the month. How is it applied?

The answer rests in the basic difference between the Hebrew year and the Roman (Julian) year. Common years in the Hebrew calendar have 353, 354, or 355 days, whereas a Roman year has 365 1/4 days. Compared to the Julian year, the Hebrew common year falls short. A Hebrew intercalary year (383, 384, or 385 days) is longer than the Roman year.

Exactly how much will the Hebrew calendar trail the Roman calendar in a common year of 12 months? The average common year is 12 times (29 d 12 h 793 p). This is 354 days, 8 hours and 876 parts.

```
            365 d   6 h    0 parts (average Julian year)
       -  ( 354 d   8 h  876 p     (average common year)
          --------------------------------
              10 d  21 h  204 p
```

In other words, the average common year of the Hebrew calendar is 10 d 21 h 204 p LESS than an average Julian year. For calculation purposes, remember the number as -10 d -21 h -204 p. This is the same as -(10 d 21 h 204 p). If you cannot recall this number under the pressures of a test, you should remember HOW it is found.

How much LONGER is an average intercalary (or leap) year than the average Julian year? To find the length of an average intercalary year, multiply 13 months times (29 d 12 h 793 p).

```
           383 d  21 h  589 p   (average leap year)
       -  (365 d   6 h    0 p)  (average Julian year)
          ----------------------------
            18 d  15 h  589 p
```

In an intercalary year, the Hebrew calendar EXCEEDS the Julian calendar by +18 d +15 h +589 p. The plus signs are carried in order to minimize confusion. As you can see, this number is very easy to find.

What about the 19 year cycle and the two calendars?

```
       29 d  12 h  793 p  (lunar months)  365 d  6 h  0 p  (Roman year)
                x  235 months per cycle          x   19 years
     -------------------                ----------------
     6939 d  16 h  595 p                6939 d 18 h  0 p
    (length of 235 lunar months)       (length of 19 Roman years)
```

How much shorter is the 19 year cycle of 235 lunar months than 19 Julian years?

```
           6939 d  18 h    0 p       19 Julian years
        - (6939 d  16 h  595 p)     235 average lunations
          ----------------------
                    1 h  485 p
```

Express this as -1 h -485 p. A 19 year cycle is one hour and 485 parts LESS than 19 Julian years.

Let's summarize these three important numbers. Be sure you understand exactly what each means!

```
---------------------------------------------------------------------

     0 d  -  1 h  - 485 p      19 year cycle is SHORTER than 19 Julian
                                  years.

  - 10 d  - 21 h  - 204 p      average common year is SHORTER than an
                                  average Julian year.

   +18 d  + 15 h  + 589 p      average leap year is LONGER than an
                                  average Julian year.

---------------------------------------------------------------------
```

You use these three numbers quite like the three other numbers you worked with in calculating the day of the week for the molad Tishri. The only complication is that some of these numbers are negative, and you must be certain that you do not overlook a negative number by assuming it is positive. It's safer and surer to carry the signs along through every step.

Now let's illustrate a calculation of the day of the month for the molad Tishri. Be aware of the fact that the years called for in this program avoid certain complications, which will be explained in program 5.

What is the day of the month of the molad Tishri in 1520 AD? Proceed as you would for finding the day of the week.

```
  -3761   BC
 -(1520)  AD                                      277 cycles elapsed
-------------                                   -------
  -5281;  5281 - 1 =  5280 years elapsed     19 / 5280 years
                                                  xxx
                                                -------
                                                   17;   3, 6, 8, 11,
                                                         14, 17 are
                                                         leap years.
                                                         6 leap years;
                                                     17  -  6  =  11
                                                      common years.
```

For the day of the month, you are determining how far the Hebrew calendar lags or leads the Roman calendar:

```
      - 1 h     485 p   (lag per 19 yr cycle)   -10 d   -21 h   -204 p
          x     277 cycles                             x   11 com. yrs.
     --------------                             ------------------------
               3395                               10      21      204
              3395                               10      21      204
              970                               ------------------------
     --------------                             -110 d  -231 h  -2244 p
     -277 h - 134345 p

                                   + 18 d  +15 h  + 589 p
                                                 x   6 leap years
                                   ------------------------------
                                  + 108 d  +90 h  +3534 p
```

The amount the Hebrew calendar will LAG behind the Roman during the elapsed time is the sum of the amounts trailed in the common years and the 19-year cycles:

```
     -110 d    -231 h    -  2244 p
               -277 h    -134345 p
     ----------------------------------
     -110 d    -508 h    -136589 p   Amt. Hebrew calendar LAGS Roman
                                        calendar during elapsed time.

     +108 d    + 90 h    +  3534 p   Amt. Hebrew calendar leads Roman
                                        calendar during elapsed time.
     ---------------------------------
       -2 d    -418 h    -133055 p


                                         -22 d                    -123 h
```

```
                       --------           ------------
                   24 / -541 h        1080 / -133,055 p
                         xxx                 xxx
                       --------           ------------
    -24 d                -13 h                  -215 p
```

-24 d -13 h -215 p represents the TOTAL AMOUNT that the Hebrew calendar trails the Roman calendar. (Just reduce the number to a convenient negative form; full reduction isn't necessary.) NEVER divide this number by 7! To find the day of the month, add the lag you have calculated to the bench mark. Since September has 30 days, you can express Oct. 6 as Sept. 36.

```
September  36    23 h  204 p       September  36    22 h  1284 p
         +(24 d -13 h -215 p)      or        -24 d -13 h  -215 p
-----------------------------      ------------------------------
                                   September  12  +  9 h +1069 p
```

The molad Tishri in 1520 AD was on September 12, just before 10 AM.

The procedure for finding the day of the month for the molad Tishri is almost exactly parallel to that for finding the day of the week. To give you an overview of both calculations, here in schematic form are the steps. Read this chart from left to right, as well as down the page.

```
------------------------------------------------------------------
Molad Tishri             Day of the week          Day of the month
------------------------------------------------------------------
Bench mark & required    3761 BC - required yr    3761 BC - required yr
year. Going from BC      (-1)?                    (-1?)
to AD? Elapsed time      ---------------------    ---------------------
in years: (divide by
19)

number of 19 yr cycles   times (2d 16h 595p)      times    (-1h -485p)
number of common years   times (4d  8h 876p)       " (-10d -21h -204p)
number of leap years     times (5d 21h 589p)       " (+18d +15h +589p)
                         -------------------      ---------------------
                         Sum is the molad "ad-    Sum is the amount
                         vancement," or excess    that the Hebrew
                         over full weeks, in the  calendar lags (-) or
                         elapsed time. (Divide    leads (+) the Roman
                         the REDUCED days by 7.)  calendar. (Reduce as
                                                  necessary.)

Add to the bench mark    +(1 d 23 h 204 p)   Oct. 6 (Sept 36) 23h 204p

                                         [Corrections, in program 5]

Answer:                  (* day of week)          (* day of month)
                         ---------------------    --------------------
------------------------------------------------------------------
```

In most cases, you will determine the day of the week first, as there is less chance for computational errors. This calculation will give you a certain number of hours and parts besides the day of the week.

Is there any way of knowing that your calculations are correct? Yes! The hours and parts for the day of the week of the molad Tishri must be identical to the hours and parts for the day of the month. For 1520 AD, the example above, the hours and parts for the day of the week are also 9h 1069p. Remember this rule for checking your work.

Since the format involved in calculating the day of the month is so very similar to the day of the week, a large amount of additional practice will not be necessary. You should, however, work out a few problems.

3.1 What is the day of the month of the molad Tishri for 1492 AD?

3.2 What is the day of the month of the molad Tishri for 28 AD?

The problems are partially worked out for you below.

3.1 1492 AD is 5252 elapsed years. This is 276 19 year cycles, and 8 elapsed years. Therefore there are 3 leap years (3, 6, 8) and 5 common years. The molad was on September 21, 19 hours and 1011 parts. (This was a Friday, in case you want to check that calculation, too.)

3.2 28 AD is 3788 elapsed years. 3788 years is 199 19 year cycles and 7 years. In 7 elapsed years, there are 3 leap years (2, 5, 7) and four common years. (If you noticed that these multipliers for the leap and common years are the same as in 3.1, you saved yourself some work!) The molad was on October 7, 8 hours 760 parts. (This was a Thursday.)

Now that you have almost completed this program, you should feel confident of your ability to calculate the day of the month for the molad Tishri. If you experienced a bit of difficulty with problems 3.1 and 3.2, you may want to work out 1520 AD again, and then compare each of your steps with the example already worked above.

3.3 As a final self-test, without consulting the program or your notes, determine the day of the month of the molad Tishri for 1448 AD. The answer is worked out in detail below.

Calculate the day of the month of the molad Tishri for 1448 AD:

```
-3761
-1448                              274 cycles
-------                          -------
 5209;  5208 elapsed years    19 / 5208
                                  xxx
                                 -------
                                    2 years; no leap years
                                            2 common

                              -10 d  21 h  -204 p
                                           x 2 common
                              --------------------
```

```
              -1 h    -485 p       -20 d  42 h  -408 p
                       274 cycles
             ----------------
               4      1940
              7      3395
             2       970
          -------------------
            -274 h -132890 p
     -20 d   -42 h    -408 p
     ------------------------
     -20 d  -316 h -133298 p  (no leap years)


                           18 h                 -123 h
                        --------            ------------
          -20 d       24 / -439 h       1080 / -133298 p
          -18 d             xxx                 xxx
          ------        ------              ------------
          -38 d            -7 h               - 458 p
     Sept  36            + 23 h               + 204 p (bench mark)
           -----------------------------------------------------------
     Sept  -2            + 16 h               - 254 p
     Sept  -2            + 15 h                 826 p
     ==========


     Sept    0    =    August 31
     Sept  - 1    =    August 30
     Sept  - 2    =    AUGUST 29   day of the month
                       =========
```

----------###----------

PROGRAM IV

USING TABLES TO FIND THE MOLAD TISHRI

PROGRAM GOAL 4

Given a table of reduced days; hours, and parts, for the Hebrew calendar, you will correctly calculate the day of the week and the day of the Roman month for the molad Tishri of selected years. Use of the table will considerably shorten the time needed.

Having worked out the problems in the first three programs of this series, you may have decided that Hebrew calendar calculations are more tedious than difficult.

Yes, some of the arithmetical operations tend to be time consuming. Perhaps you feel that you can calculate the molad Tishri quite well, but you'd prefer having some kind of a desk top computer just to save time and frustration! Not that all the steps are too involved for you to do yourself; only the lengthy multiplication and division.

Finding the elapsed time is just a quick subtraction or addition operation. Expressing the elapsed time in terms of 19 year cycles still isn't demanding. The real hang-up comes in the multiplication of the 19 year cycles, the leap years, and the common years, right?

Most of that step, so far as the multiplication is concerned, can be eliminated by using a table of reduced days, hours, and parts. You will be able to calculate the molad Tishri in less than half the time previously required!

Examine part I of the chart included in this program. On the left side of the page as you read it is a table marked "19 YEAR TIME CYCLES." Toward the bottom of the page are two other tables "INTERCALARY YEARS" and "COMMON YEARS." (We'll return to the other table on part I a little later.)

As you suspect, each of these three tables will replace the multiplication operations you did before in order to find the excess over a number of full weeks, or the molad "advancement." Now you can read a reduced number from the table very conveniently!

Suppose you have a problem in which the elapsed time is 200 19 year cycles, three leap (intercalary) years, and five common years. What will be the excess over full weeks?

200	19 year cycles:	5 d	22 h	200 p
3	leap years:	3 d	16 h	687 p
5	common years:	0 d	20 h	60 p
Total:		8 d	48 h	947 p
	Excess over full weeks:	2 d	0 h	947 p

Of course, most of your calculations of elapsed time will involve intermediate values of 19 year cycles, which are not directly on the first table. What do you do with 189 cycles, 315 cycles, and the like? Just add them with the units you already have. If, in the example above, we had 276 19 year time cycles instead of 200, you would add the molad "advancement" for 70 cycles and for 6 cycles:

	200	19 year cycles:	5 d	22 h	200 p
276	70	19 year cycles:	6 d	6 h	610 p
cycles	6	19 year cycles:	2 d	3 h	330 p
	3	leap years:	3 d	16 h	687 p
	5	common years:	0 d	20 h	60 p
	Total:		16 d	67 h	1887 p

You can add these numbers in less time than it takes to multiply 276 x (2 d 16 h 595 p). Notice that you can perform all your addition in one step.

For the 19 year cycles, it may be easier in some problems to do a subtraction of two numbers in the table rather than an addition of three. The excess over full weeks of 297 19 year cycles can be found either way:

```
add    200:      5 d  22 h  200 p
        90:      4 d   1 h  630 p
         7:      4 d  19 h  925 p
-----------------------------------
297 cycles:     13 d  42 h 1755 p
reduced:         0 d  19 h  675 p


subtract 300:    1 d  21 h  300 p (1 d 20 h 1380 p)
          -3:   -1 d  -1 h -705 p
-----------------------------------
                 0 d  19 h  655 p
```

Now practice using the table to find the day of the week of the molad Tishri for:

4.1 1520 AD. The problem is worked out below.

```
3761 BC
1520 AD                                 277 cycles
-----------                            -------
5281;     5280 elapsed years       19 / 5280
                                        xxx
                                       -------
                                          17 yrs: 3, 6, 8,
                                             11, 14, 17
                                             6 leap
                                             11 common
```

Using the table:

```
200 cycles:      5 d  22 h  200 p
 70     "        6 d   6 h  610 p
  7     "        4 d  19 h  925 p
  6 leap:        0 d   9 h  294 p
 11 common:      6 d   0 h  996 p
-----------------------------------
Total:          21 d  56 h 3025 p
Excess over full
     weeks:      2 d  10 h  865 p
bench mark    +( 1 d  23 h  204 p)
------------------------------------
Reduced          3 d  33 h 1069 p
                 4 d   9 h 1069 p
```

Molad Tishri: Wednesday 1520 AD

The first time through a calculation with the chart took more time than you will need later. But we can even go through a problem faster by using the table on the first page called "ELAPSED YEARS IN ONE 19 YEAR CYCLE."

This table combines the leap years and the common years of a 19 year cycle together, both for dates before 142 AD and for those after. Once you divide the elapsed time by 19, you can match the remainder directly without figuring leap years and common years separately. If your remainder is 12 (for a year after 142 AD), the excess over full weeks would be 2 d 12 h 724 p.

4.2 Re-calculate the day of the week for the molad Tishri for 1520 AD using the combined table. The answer is below.

```
          3761 BC
          1520 AD                           277 cycles
          ----------                      -------
          5281; 5280 years            19 / 5280 years
                                           xxx
                                          -------
                                             17 elapsed years
```

From the chart, part I:

```
          200  cycles:          5 d     22 h   200 p
           70      "            6 d      6 h   610 p
            7      "            4 d     19 h   925 p
           17  elapsed years:   6 d     10 h   210 p
          -------------------------------------------
          Total                21 d     57 h  1945 p
          bench mark            1 d     23 h   204 p
                         ----------------------------
                               22 d     80 h  2149 p
          Reduced:              4 d      9 h  1069 p
```

Molad Tishri: Wednesday.

Notice that you need not reduce until the last step. If you want, it's even possible to add the benchmark after the elapsed years of a cycle-without sub-totaling first. Once you thoroughly understand the concepts involved in calculating the molad Tishri, you can make these economies in your work.

(Incidentally, if you choose to add the bench mark along with the numbers from the table, your answer -- after reduction -- may take the form of 0d   xx h   xxx p. What day of the week is 0 d? Just add 7 to 0 d, and you will have the real day -- the sabbath. When you are working with the days of the week, "7" is the "additive complement" of numbers from -6 to 0. There's nothing mysterious here, just a mathematical "law".)

Now look at part II of the Hebrew calendar chart. This section will assist you in calculating the day of the month. The tables are very much like those of part I. The negative numbers represent the amount the Hebrew calendar lags behind the Roman calendar for a given unit of time. The positive numbers indicate that the Hebrew calendar leads the Roman.

Because of the different signs involved, you'll find it easier to total the negative numbers together before combining the positive. For example, how much does the Hebrew calendar trail the Roman after 276 19 year cycles plus 8 elapsed years?

```
          200 cycles:    -12 d     -  1 h     - 880 p
          70 cycles:     - 4 d     -  5 h     - 470 p
```

```
6 cycles:      - 0 d     -  8 h     - 750 p
--------------------------------------------
276 cycles:    -16 d     - 14 h    - 2100 p
8 elapsed yrs. + 2 d     - 12 h    +  747 p
--------------------------------------------
Total:         -14 d     - 26 h    - 1353 p
```

All you need do now is add the bench mark. The Hebrew calendar is usually behind the Roman, so the total will be mostly negative numbers. The parts or hours COULD be positive in some problems. Watch your SCRIBAL ACCURACY!

```
Sept 36      23 h      204 p  (bench mark)
    -14 d   -26 h    -1353 p
-----------------------------
```

Borrowing:

```
Sept 35      45 h     2364 p
    -14 d   -26 h     1353 p
-----------------------------
Sept 21      19 h     1011 p
```

Whenever you are making a calculation, it will be faster to use the combined table instead of the individual tables of intercalary years and elapsed years. But you can take either option.

If you find that calculations of the day of the month are different from those of the day of the week, be sure that you used the right table for the right number!

4.3 Calculate with the use of the chart the day of the month of molad Tishri for 1520 AD. The answer is below.

```
1520 AD
3761 BC                              277 cycles
------------                       -------
5281; 5280 years               19 / 5280
                                    xxx
                                   -------
                                      17 elapsed years
```

From the chart, part II:

```
200 cycles:      -12 d   - 1 h   - 880 p
 70 cycles:      - 4 d   - 5 h   - 470 p
  7 cycles:      - 0 d   -10 h   - 155 p
-------------------------------------------
                - 16 d   -16 h   -1505 p
17 years        -  7 d   -20 h   + 210 p
--------------------------------------------
                - 23 d   -36 h   -1295 p
bench mark  Sept. 35      45 h    2364 p  (borrowing)
--------------------------------------------
            Sept. 12 d     9 h    1069 p
```

You can set up both the day of the week and the day of the month calculations side by side to save space if you like.

4.4 Test yourself on a complete calculation of the molad Tishri, day of the week and the Roman date, for 1000 AD. Consult the chart in an efficient manner.

```
          1000 AD
          3761 BC                              250 cycles
          -----------                        -------
          4761; 4760 years              19 / 4760
                                             xxx
                                            -------
                                               10 elapsed years

          Day of the week                    Day of the Roman month

     200 cycles:   5 d   22 h   200 p        -12 d  -  1 h  -  880 p
      50 cycles:   1 d   11 h   290 p        - 3 d  -  0 h  -  490 p
                                             -------------------------
  10 elapsed yrs.  6 d    6 h   339 p        -15 d  -  1 h  - 1370 p
  bench mark:      1 d   23 h   204 p        -20 d  -  6 h  +  339 p
  -----------------------------------        -------------------------
        Total:   13 d   62 h  1333 p         -35 d  -  7 h  -  931 p
      Reduced:    1 d   15 h   253 p   Sept  36    + 22 h  + 1284 p
                                             -------------------------
                                               1 d    15 h     253 p
```

The hours and the parts agree.
In 1000 AD the molad Tishri was Sunday, Sept. 1, 15 h 253 p.

By using the Hebrew calendar chart, notice how much less space on the page a complete calculation of the molad Tishri now takes. And easier, isn't it!

THE HEBREW CALENDAR

Tables of Reduced Days, Hours, and Parts for 19 Year Time Cycles, Intercalary Years, and Common Years.

I. Advancement of the Molad Over a Full Number of Weeks

19 YEAR TIME CYCLES

Elapsed Cycles	Excess over full weeks		
1	2d	16h	595p
2	5	9	110
3	1	1	705
4	3	18	220
5	6	10	815
6	2	3	330
7	4	19	925
8	0	12	440
9	3	4	1035
10	5	21	550

20	4	19	20
30	3	16	570
40	2	14	40
50	1	11	590
60	0	9	60
70	6	6	610
80	5	4	80
90	4	1	630
100	2	23	100
200	5	22	200
300	1	21	300

----------###----------

INTERCALARY YEARS

Elapsed Yrs.	Excess over full weeks		
1	5d	21h	589p
2	4	19	98
3	3	16	687
4	2	14	196
5	1	11	785
6	0	9	294
7	6	6	883

----------###----------

ELAPSED YEARS IN ONE 19 YEAR TIME CYCLE

Elapsed Years	Before 142 AD Excess over full weeks			142 AD and after Excess over full weeks			Elapsed Years
1	4d	8h	876p	4d	8h	876p	1
2	3	6	385	1	17	672	2
3	0	15	181	0	15	181	3
4	4	23	1057	4	23	1057	4
5	3	21	566	2	8	853	5
6	1	6	362	1	6	362	6
7	0	3	951	5	15	158	7
8	4	12	747	4	12	747	8
9	1	21	543	1	21	543	9
10	0	19	52	6	6	339	10
11	5	3	928	5	3	928	11
12	2	12	724	2	12	724	12
13	1	10	233	6	21	520	13
14	5	19	29	5	19	29	14
15	3	3	905	3	3	905	15
16	2	1	414	0	12	701	16
17	6	10	210	6	10	210	17
18	5	7	799	3	19	6	18

----------###----------

COMMON YEARS

Elapsed Yrs.	Excess over full weeks		
1	4d	8h	876p
2	1	17	672
3	6	2	468
4	3	11	264
5	0	20	60
6	5	4	936
7	2	13	732
8	6	22	528
9	4	7	324
10	1	16	120
11	6	0	996
12	3	9	792

II. Time Differences

19 YEAR TIME CYCLES

Elapsed Cycles	Time Difference		
1	- 0d	- 1h	- 485p
2	- 0	- 2	- 970
3	- 0	- 4	- 375
4	- 0	- 5	- 860
5	- 0	- 7	- 265
6	- 0	- 8	- 750
7	- 0	-10	- 155
8	- 0	-11	- 640
9	- 0	-13	- 45
10	- 0	-14	- 530
20	- 1	- 4	-1060
30	- 1	-19	- 510
40	- 2	- 9	-1040
50	- 3	- 0	- 490
60	- 3	-14	-1020
70	- 4	- 5	- 470
80	- 4	-19	-1000
90	- 5	-10	- 450
100	- 6	- 0	- 980
200	-12	- 1	- 880
300	-18	- 2	- 780

----------###----------

INTERCALARY YEARS

Elapsed Inc. Years	Time Difference		
1	+ 18d	+15h	+589p
2	+ 37	+ 7	+ 98
3	+ 55	+22	+687
4	+ 74	+14	+196
5	+ 93	+ 5	+785

6	+111	+21	+294
7	+130	+12	+883

----------###----------

ELAPSED YEARS IN ONE 19 YEAR TIME CYCLE

Elapsed Years	Before 142 AD Time Difference			142 AD and after Time Difference			Elapsed Years
1	-10d	-21h	-204p	-10d	-21h	-204p	1
2	+ 8	- 6	+385	-21	-18	-408	2
3	- 3	- 3	+181	- 3	- 3	+181	3
4	-14	0	- 23	-14	0	- 23	4
5	+ 5	- 9	+566	-24	-21	-227	5
6	- 6	- 6	+362	- 6	- 6	+362	6
7	+13	-15	+951	-17	- 3	+158	7
8	+ 2	-12	+747	+ 2	-12	+747	8
9	- 9	- 9	+543	- 9	- 9	+543	9
10	+10	-17	+ 52	-20	- 6	+339	10
11	- 1	-15	+928	- 1	-15	+928	11
12	-12	-12	+724	-12	-12	+724	12
13	+ 7	-20	+233	-23	- 9	+520	13
14	- 4	-17	+ 29	- 4	-17	+ 29	14
15	-15	-15	+905	-15	-15	+905	15
16	+ 4	-23	+414	-26	-12	+701	16
17	- 7	-20	+210	- 7	-20	+210	17
18	+11	- 5	+799	-18	-17	+ 6	18

----------###----------

COMMON YEARS

Elapsed Comm. Years	Time Difference		
1	- 10d	-21h	- 204p
2	- 21	-18	- 408
3	- 32	-15	- 612
4	- 43	-12	- 816
5	- 54	- 9	-1020
6	- 65	- 7	- 144
7	- 76	- 4	- 348
8	- 87	- 1	- 552
9	- 97	-22	- 756
10	-108	-19	- 960
11	-119	-17	- 84
12	-130	-14	- 288

----------###----------

as they are often different; e. g. 1 positive and 2 negative.

- signifies that the Hebrew calendar is behind, or trails, the Julian calendar.

+ signifies that the Hebrew calendar is ahead of, or leads, the Julian calendar.

PROGRAM V

MAKING ROMAN LEAP YEAR AND JULIAN / GREGORIAN CORRECTIONS

PERFORMANCE GOAL 5

For any required Roman year, AD or BC, you will correctly make the necessary adjustments for:

a) the Roman leap year / common year pattern
b) the conversion from the Julian to the Gregorian calendar

In your day of the month calculations of the molad Tishri.

By actively working through the first four programs, you've learned how to calculate the molad Tishri for the specified years. Now you will apply two corrections to the DAY OF THE MONTH calculations. This will extend your ability to find the molad of Tishri to virtually any Roman year required, whether AD or BC. Keep in mind that these corrections affect ONLY the day of the month calculations! The day of the week part of your work needs no adjustment.

The first correction involves the length of a Roman year. On a calendar, a Roman year has either 365 days (common year) or 366 days (leap year). In your calculations for the day of the month, you used the average length of a Roman year, 365 1/4 days. How do you make allowance for the difference?

You add six hours to the molad (day of the month) for every year after a Roman leap year:

If the required Roman year is leap, add 0 hours.
If the required Roman year is one year after a leap year, add 6 hours.
If the required Roman year is two years after a leap year, add 12 hours.
If the required Roman year is three years after a leap year, add 18 hours.

(Don't become confused by what a "leap year" means. A Hebrew leap year has 13 months; a Roman leap year has 366 days. The context will tell you what applies.)

How can you know if the required Roman year is leap or common? Simply divide the year by four and note the remainder: 10 AD divided by 4 gives a remainder of two; 51 BC divided by 4 gives a remainder of three; 1977 AD divided by 4 leaves a remainder of one.

For AD years, the remainder numerically corresponds to the Roman leap year/common year pattern:

A remainder of 0 signifies that the required Roman year is leap.

"	1	"	one after a leap.
"	2	"	two after a leap.
"	3	"	three after a leap.

BC years have a different pattern of remainders. Since 4 AD is a leap year, 1 AD is three years before a leap year. 1 BC is a leap year, too, being four years before 4 AD. Four years before 1 BC is 5 BC, a leap year. And 9 BC is a leap year. Then 8 BC is one year after a Leap year; 7 BC is two years after; 6 BC is three after. If you divide these years by 4, you can find the remainder that corresponds to a leap year (9 divided by 4 leaves a remainder of 1), a year after a leap year, etc. This pattern of remainders from 9 BC to 6 BC will be valid for years further back into antiquity.

The chart below summarizes the remainder patterns for both BC and AD Roman years.

Roman leap year corrections (dividing the required year by 4)

==

AD:

If the remainder is 0, add no hours to the day of month calculations

"	1	6 hours
"	2	12 hours
"	3	18 hours

BC:

If the remainder is 1, add no hours to the day of month calculations

"	0	6 hours
"	3	12 hours
"	2	18 hours

To insure that you can apply this correction to the day of the month, indicate how many hours you would add for each of the following years.

5.1	1520 AD	5.10	1917 BC
5.2	1974 AD	5.11	4 BC
5.3	1983 AD	5.12	971 BC
5.4	1871 AD	5.13	1181 BC
5.5	1699 AD	5.14	1020 BC
5.6	31 AD	5.15	1984 AD
5.7	33 AD	5.16	1020 AD
5.8	142 AD	5.17	2001 BC
5.9	1486 BC	5.18	2001 AD

The answers appear below.

5.1	0 hours	5.7	6 hours	5.13	0 hours
5.2	12 hours	5.8	12 hours	5.14	6 hours
5.3	18 hours	5.9	18 hours	5.15	0 hours
5.4	18 hours	5.10	0 hours	5.16	0 hours
5.5	18 hours	5.11	6 hours	5.17	0 hours
5.6	18 hours	5.12	12 hours	5.18	6 hours

When you are checking your calculations for the molad of Tishri, you may find that the day of the month calculation differs from the day of the week by six hours, twelve hours, or eighteen hours. Here's what has happened: You forgot to make the Roman leap year correction in the day of the month calculation! Divide the Roman year by 4 and inspect the remainder on EVERY calculation involving the day of the month.

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The second correction to the day of the month calculation of the molad Tishri concerns the corresponding Gregorian calendar date of a day on the Julian calendar. By understanding the reasons for the change from the Julian calendar to the Gregorian, you will have no difficulty in performing this correction.

The Julian calendar was named after Julius Caesar; the Gregorian calendar after Pope Gregory XIII. Both calendars are "Roman." The Gregorian calendar is what you use every day.

When did the Roman calendar, based upon an average yearly length of 365 1/4 days, come into use? The answer is 45 BC. With the aid of the Egyptian astronomer Sosigenes, Julius Caesar completely revised the previous lunisolar calendar, which had drifted badly with respect to the seasons. To effect the reform, "46 BC" had 445 days assigned to it in order to correct for all the previous errors. That year was very appropriately called the "year of confusion!" In 45 BC the vernal (spring) equinox occurred on March 25.

The Julian calendar wasn't without its faults, however. The average Julian year was eleven minutes and fourteen seconds LONGER than a "tropical year." (A tropical year is measured from one vernal equinox to the next.) After 128 years the Julian calendar had an extra day, compared to an equal number of tropical years.

By the time of the famous Council of Nicea in 325 AD, the Julian calendar was about three days behind the tropical year. This meant that the vernal equinox was several days "early" on the Julian calendar. Accordingly, the churchmen based their rules for the date of Easter on the spring equinox falling on March 21 of the Julian calendar. It had been March 25 in 45 BC.

Towards the end of the Counter-Reformation in the sixteenth century, the spring equinox had "crept back" to about March 11. To alleviate the problem, Pope Gregory reformed the Julian calendar by deleting ten days from the month of October 1582: The day after October 4 officially became October 15. So after 1582 the spring equinox shifted back to March 21--where it had been during the time of Constantine the Great. (Of course, the equinox never shifted; the calendars did the moving!)

Pope Gregory also invoked a new rule concerning Roman leap years. The Julian calendar considers every fourth year as having 366 days. To shorten the new Roman calendar by three days in 400 years, the Pope declared that century years NOT EVENLY DIVISIBLE BY 400 would remain COMMON (no February 29).

Therefore, 1700 AD, 1800 AD, and 1900 AD, which are not evenly divisible by 400, were common years according to the Gregorian calendar rules. By the old Julian calendar, they would have been leap years.

Instead of the eleven minute error in the average Julian year, the average Gregorian year is only 26 seconds too long.

How do you convert a Julian date into a day on the Gregorian calendar? Just add the TOTAL DAYS that have been dropped from the Julian! Before 1582 AD no correction is needed, as all the Roman dates are understood to be Julian. But after 1582 AD, you must add at least ten days to the Julian day of the month calculation for the molad Tishri.

Bear in mind that 1600 AD was a leap year in both the Julian and the Gregorian calendars. The difference between the calendars remained ten days until 1700. The Gregorian calendar omitted February 29 that year (because the year wasn't evenly divisible by 400), while the Julian retained the extra day. From 1700 to 1799, the Julian calendar was eleven days behind the Gregorian since that was the total number of days dropped.

In the same manner, from 1800 to 1899 you must add twelve days to the Julian date to find the corresponding Gregorian day. During the 1900's and 2000's, you add thirteen days.

How many days will you add to your day of the month calculations of the molad of Tishri to convert from the Julian calendar to the Gregorian calendar for each of the following years? (Before 1582 AD, simply respond "0 d".)

5.19 1520 AD

5.20 1601 AD

5.21 1798 AD

5.22 1914 AD

5.23 1851 AD

5.24 1666 AD

5.25 1583 AD

5.26 1984 AD

5.27 2001 AD

5.28 2145 AD

5.29 1979 AD

5.30 1712 AD

The answers appear below.

5.19 + 0 d	5.25 +10 d
5.20 +10 d	5.26 +13 d
5.21 +11 d	5.27 +13 d
5.22 +13 d	5.28 +14 d
5.23 +12 d	5.29 +13 d
5.24 +10 d	5.30 +11 d

* * *

Neither of the corrections to the day of the month calculation of the molad Tishri is complicated. However, you must remember to perform them whenever the Roman year calls for such adjustments. The last problem in this program will challenge you to put together all the calendar skills you have learned thus far.

5.31 What is the day of the week and the date of the Roman month for the molad Tishri in 2055 AD? (You may use the chart of reduced numbers.) This problem is worked out in detail for you on the next page.

What is the day of the week and the date of the Roman month for the molad of Tishri in 2055 AD?

```
A.   The ELAPSED TIME is:     - 3761
                              - 2055
                              -------
                              - 5816 - subtract 1 going from BC to AD:
                                          5815 years.
```

B. 5815 years is 5815 / 19 nineteen year cycles.

```
                          306
                         -------
                    19 / 5815      306 cycles and the first year of the
                         57        next cycle have elapsed.
                         ------
                          115
                          114
                         ------
                            1
```

C. THE DAY OF THE WEEK FOR THE MOLAD TISHRI:

From the tables, find the advancement of the molad.

```
300  19-year cycles:          1 d  21 h   300 p
  6  19-year cycles:          2 d   3 h   330 p
  1 year of next cycle:       4 d   8 h   876 p
                              ------------------
                              7 d  32 h  1506 p
     Reducing this:                +1 h -1080 p
                              ------------------
                              7 d  33 h   426 p
                            + 1 d -24 h
                              ------------------
     The advancement is:      1 d   9 h   426 p   over full number
                                                  of weeks.
     Add the advancement to
     the bench mark:        +(1 d  23 h   204 p)
                              ------------------
                              2 d  32 h   630 p

     This reduces to:         3 d   8 h   630 p
```

The molad was on the third day of the week, Tuesday.

D. THE DAY OF THE ROMAN MONTH:

From the tables, find how far behind the Roman calendar the Hebrew calendar is:

```
300   19-year cycles:    -18 d  - 2 h  - 780 p
  6   19-year cycles:    - 0 d  - 8 h  - 750 p
  1 year of next cycle:  -10 d  -21 h  - 204 p
                         --------------------------
                         -28 d  -31 h  -1734 p
The benchmark is:      Sept. 36  23 h    204 p
```

"Borrow" 2 x 1080 parts, and one day:

```
                       Sept. 35  45 h   2364 p
```

Add the time difference in the calendars to the benchmark:

```
                       +(-28 d  -31 h -1734 p)
               --------------------------------
                       Sept.  7  14 h   630 p
```

Two corrections must be made, that of the ROMAN leap year, and the Julian / Gregorian:

```
      513
     -------
 4 / 2055
     20
     -------
      5             3 years after a Roman leap year means
      4             you must add 18 HOURS.
```

```
           ---
           15
           12
           ---
            3
```

During the 1900's, the correction to the Julian calendar is 13 days. Since 2000 is evenly divisible by 400, the correction during the 21st century is STILL 13 days.

```
          Sept. 7   14 h  630 p
                   +18 h          Roman leap year correction
               13 d                    Julian / Gregorian correction
          --------------------------
         Sept. 20   32 h  630 p
Molad
Tishri:  Sept. 21    8 h  630 p    parts and hours agree
                                   with day of week
```

----------###----------

PROGRAM VI

APPLYING THE POSTPONEMENT RULES TO FIND TISHRI ONE

PERFORMANCE GOAL 6A: Given the molad Tishri for a required year, you will correctly determine the day of the week and the day of the month of the Feast of Trumpets, Tishri 1, by applying from memory the four postponement rules explained in this program.

Example: In 1987, the molad of Tishri will be:

September 23 Wednesday (4 d) 3 h 77 p

What is the date for the Feast of Trumpets?

Answer: September 24 Thursday (by rule two)

PERFORMANCE GOAL 6B: Without consulting notes, you will correctly explain (in a brief paragraph for each rule) why each postponement for Tishri 1 is important for the Hebrew calendar.

PERFORMANCE GOAL 6A: After correctly calculating both the day of the week and the day of the month for the molad of Tishri the next step is to find the date of Tishri 1, the Feast of Trumpets. You will analyze the time of the molad to see how four postponement rules affect the Feast of Trumpets.

At the outset, you should understand that a conjunction of the earth, moon, and sun takes place completely apart from man's doings. The MOLAD CANNOT BE POSTPONED BY HUMAN ENACTMENTS! Tishri 1, the civil New Year in the Hebrew calendar, is what is postponed.

Here are the four postponement rules, prefaced by a general statement of the case where there is no postponement:

When the molad Tishri occurs at a time of the week that's unaffected by the four postponement rules, the Feast of Trumpets is on the SAME DAY as the molad.

Rule one: When the molad of Tishri occurs AT NOON OR LATER (12 h 0 p or more in your calculations), the Feast of Trumpets is postponed until the next day.

Rule two: When the molad of Tishri OR a postponement occurs on a Sunday, Wednesday, or Friday, the Feast of Trumpets is postponed one day, to a Monday, Thursday, or Sabbath, respectively.

Rule three: When the molad of Tishri of a COMMON YEAR is on a Tuesday, at or after (3 d) 3 h 204 p the Feast of Trumpets is postponed to a Wednesday, and by Rule two, further postponed to a Thursday.

RULE FOUR: When the molad of Tishri of a COMMON year IMMEDIATELY FOLLOWING an intercalary year occurs on a Monday, at or after (2 d) 9 h 589 p the Feast of Trumpets is postponed to a TUESDAY.

You should take notice that the maximum postponement of Tishri 1 is two days. Also realize that the order of these rules is significant. They are easiest to apply in the numerical order above. Rule one governs all afternoon time periods, so rules three and four ONLY AFFECT A PORTION OF THE MORNINGS.

You will quickly discover that rules one and two are by far the most frequently used, both individually and together -- one, two, or one and two. And years where no postponement of Tishri 1 is required are rather common, too.

This means, of course, that rules three and four are quite infrequent, especially the last one. Rule three has rule two "built in", since it is a very specific case. Therefore, rule three and rule four are always separately used. Whenever rules one and two apply, either separately or in combination, you know in advance that rules three and four will not be involved. All this speeds up your use of these rules considerably.

Before you see examples of each rule applied, one more thing needs to be explained. How do you know when the year in question is common, and when does it immediately follow a leap year?

Remember how you determined elapsed time by adding (or subtracting) the required year from the bench mark? Well, the year in question is the year AFTER the number of elapsed years. The distinction between the terms "elapsed years" and "required year" is quite similar to your age. All through your 21st year of life, your age is twenty!

What do you do to find which year of the cycle is a required year? Add 1 to the remainder you already found when you divided the elapsed years by 19. This new remainder will be the number of the year in the cycle. If the remainder for elapsed years is 10, the required year is year 11 of the l9-year cycle.

You will recall that for the years 142 AD (handwritten note: 256

AD -- Dr. Hoeh correct date) and after, the intercalary years are 3, 6, 8, 11, 14, 17, 19. The years immediately following these years are the ones you are concerned with for rule four: 4, 7, 9, 12, etc. All the years in the cycle which aren't intercalary are common.

Whenever you are working with years earlier than 142 AD, be sure to use the proper intercalary years, which are one year earlier than the present cycle: 2, 5, 7, 10, 13, 16, 18. The year after these leap years will then apply for rule four.

Now you will learn how to analyze the molad Tishri to apply the postponement rules.

1492 AD: Molad Tishri was September 21, Friday 6 d 9 h 1011 p.

19 h 1011 p is after 12 h 0 p; RULE ONE postpones to the Sabbath. The Sabbath is a permissible day; no further postponement.

Feast of Trumpets was September 22, Sabbath.

1584 AD: molad Tishri was September 5, Wednesday 4 d 2 h 852 p.

2 h 852 p is before 12 h 0 p; rule one not involved. Wednesday is a forbidden day; RULE TWO applies.

Feast of Trumpets was September 6, Thursday.

1615 AD: molad Tishri was September 22, Tuesday 3 d 20 h 804 p.

20 h 804 p is after 12 h 0 p; RULE ONE postpones to Wednesday. Wednesday is a forbidden day; RULE TWO re-postpones to Thursday.

Trumpets was September 24, Thursday.

1632 AD: molad Tishri was September 14, Tuesday, 3 d 6 h 1014 p.

6 h 1014 p is before 12 h 0 p. Rule one doesn't apply. Tuesday is a permissible day. Rule two doesn't apply. 6 h 1014 p is after 3 h 204 p. Rule three could be involved. Is 1632 a common year?

The elapsed time from 3761 BC is 5392 years to 1632 AD. Then the year in question is the 5393rd year, or the 16th year of a cycle after 283 full cycles. This is a common year. RULE THREE does apply.

Tishri 1 is September 16, Thursday.

841 AD: molad Tishri was September 19, Monday, 2 d 11 h 735 p.

Rules one, two, and three, don't apply. The molad is on a Monday, and 11 h 735 p is after 9 h 589 p. Is 841 AD a common year immediately after a leap year? From 3761 BC to 841 AD is 4601 years; the required year is the 4602 year. This is 242 19-year cycles plus 4 years of the next. This is a common year immediately following a leap year. RULE FOUR applies.

Trumpets was September 20, Tuesday.

1910 AD: molad Tishri was October 4, Tuesday, 3 d 0 h 61 p.

0 h 61 p is before 12 h 0 p; rule one doesn't apply. Tuesday is a permissible day; rule two doesn't apply. 0 h 61 p is before 3 h 204 p; rule three doesn't apply. Rule four can't apply to Tuesday. No postponement.

Trumpets is October 4, Tuesday.

As you can see, only a few seconds are needed to check a molad of Tishri for applicable postponements. Don't under-estimate the importance of doing this operation correctly, however! All of the other Holy Days are derived from the date of Tishri one!

Practice applying the postponement rules to the following molads. The correct dates for the Feast of Trumpets are listed on the next page, along with the postponement rules which are needed.

6a.1 1264 AD September 22, Monday 2 d 13 h 351 p

6a.2 1255 AD September 3, Friday 6 d 3 h 95 p

6a.3 1259 AD September 18, Thurs. 5 d 15 h 865 p

6a.4 2001 AD September 17, Monday 2 d 22 h 106 p

6a.5 2008 AD September 30, Tuesday 3 d 1 h 1057 p

6a.6 2014 AD September 24, Wednesday 4 d 8 h 339 p

6a.7 1984 AD September 25, Tuesday 3 d 11 h 976 p

6a.8 1985 AD September 14, Sabbath 7 d 20 h 772 p

6a.9 462 AD September 10, Monday 2 d 1 h 511 p

6a.10 496 AD September 23, Monday 2 d 10 h 644 p

6a.11 134 AD October 5, Monday 2 d 23 h 343 p

6a.12 118 AD October 2, Sabbath 7 d 21 h 1009 p

6a.13 588 BC September 27, Tuesday 3 d 3 h 209 p

6a.141 953 BC September 12, Sunday 1 d 12 h 4 p

Answers on following page:

Answers:		Rule
6a.1	September 23, 1264 AD; Tuesday	1
6a.2	September 4, 1255 AD; Sabbath	2
6a.3	September 20, 1259 AD; Sabbath	1, 2

6a.4	September 18, 2001 AD; Tuesday	1
6a.5	September 30, 2008 AD; Tuesday	---
6a.6	September 25, 2014 AD; Thursday	2
6a.7	September 27, 1984 AD; Thursday	3
6a.8	September 16, 1985 AD; Monday	1, 2
6a.9	September 10, 462 AD; Monday	---
6a.10	September 24, 496 AD; Tuesday	4
6a.11	October 6, 134 AD; Tuesday	1
6a.12	October 4, 118 AD; Monday	1, 2
6a.13	September 29, 588 BC; Thursday	3
6a.14	September 13, 953 BC; Monday	1

* * * * * *

PERFORMANCE GOAL 6B: You do not need to understand the reasons behind the postponement rules in order to correctly apply them. However, your appreciation of the Sacred calendar will be enhanced by grasping the purpose of each rule.

WHY RULE ONE, NOON OR AFTER POSTPONEMENTS?

Whenever the molad of Tishri occurs at noon or after (12 h 0 p), Tishri one is postponed to the next day (at least). But how did the molad come to be associated with noon? Rule one points back to the initial formulation of the Hebrew calendar -- in the days of Seth, according to Josephus. God intended that the heavenly bodies would intrigue man to study their movements carefully. "Let there be lights in the firmament of the heaven to divide the day from the night; and let them be for signs, and for seasons, and for days, and for years" (Gen. 1:14).

As you may have observed, the sun, moon and stars rise and set each day. But the sun rises each day about four minutes later with respect to the stars. It has a proper motion of its own, independent of the stars. This motion is not due to the daily rotation of the earth on its axis, which gives the illusion that the sun moves westward each day. The apparent path of the eastward journey of the sun through the stars, known as the "ecliptic", is due to the earth's annual orbital revolution.

The earth is tilted 232 degrees on its axis. By studying the diagram below, you can see that the plane of the earth's equator is inclined 232 degrees from the plane of the earth's orbit. Astronomers call the projection of the earth's equator into space the "celestial equator".

(NOTE: To view the diagram mentioned above, see the file HEB-CAL1.TIF in the Images\OtherWCG directory.)

Seasons occur because the earth's orbital plane (the plane of the earth's orbit) is not coincident with the plane of the earth's equator. The vernal equinox takes place when the apparent northward moving sun crosses the celestial equator. It again intersects the celestial equator when the sun passes below the celestial equator moving southward--at the time of the autumnal equinox. On the days of the equinoxes, the day and night periods are equal.

Consider the next diagram, which is a closeup of the vernal equinox. The moment of the equinox occurs when the center of the sun crosses the celestial equator. But the sun's apparent diameter is 1/2 degree; therefore the TRAILING EDGE of the sun is still 1/4 degree behind the equinoctial point.

(NOTE: To view the diagram mentioned above, see the file HEB-CAL2.TIF in the Images\OtherWCG directory.)

How long will it take for the trailing edge of the sun to pass the point of the vernal equinox? The sun's eastward progress along the ecliptic is about 1 degree per day (since the earth revolves 360 degrees around the sun in about 365 days). In 24 hours the sun moves 1 degree; in 12 hours it moves 1/2 degree; in 6 HOURS it moves 1/4 degree.

Six hours before sunset is 12 noon. Unless the equinox occurs BEFORE noon, the sun's trailing edge will not have passed the celestial equator by sunset! When the equinox occurs at noon or later, Sol is still a winter sun. The first day of spring is therefore assigned to the following day.

This is one reason why noon became a logical demarcation point for time in astronomical matters. Noon is a stable observation point for time, irrespective of the observer's latitude. (The time of sunrise and sunset, but not noon, vary considerably during the course of a year, especially in the extreme latitude.) From earliest known times until 1925, astronomers had traditionally used noon as a reference point for the day. Thus noon became in antiquity a limit point not only for the equinox, but also for the molad.

Just as noon arbitrated the first day of spring and the first day of fall, it served a similar function with the assigned day of the molad -- that is, the molad or conjunction of the moon had to have a natural and arbitrary limit, in this case noon.

WHY RULE TWO, FORBIDDEN DAYS?

This postponement rule prevents Holy Days from falling on a Sunday (during the fall) and the Passover occurring at awkward times:

If Trumpets could occur on a Wednesday, the Day of Atonement, Tishri 10, would fall on a Friday, the preparation day for the Sabbath! And the Passover would also be observed Saturday night, a most difficult time.

If Trumpets could occur on a Friday, the Day of Atonement would be on a Sunday, the day after the Sabbath.

If Trumpets could occur on a Sunday, then the first day of the Feast of Tabernacles, as well as the last Great Day would be on a Sunday.

WHY RULE THREE, THE TUESDAY-COMMON YEAR POSTPONEMENT?

As you recall, the maximum length of a common year in the Hebrew calendar is 355 days. Without this rule, a common year might have 356 days!

Anytime in the morning of a Tuesday, 3h   204 p is affected by this rule. Why this particular moment?

```
Start with Tuesday        3 d  3 h  204 p
Add an average        + ( 4 d  8 h  876 p)   the excess over full
common year         -----------------------  number of weeks
                          7th 11 h 1080 p
```

This number is the same as the seventh day 12 hours and no parts. This means that the next molad Tishri would occur on a Sabbath at noon. But rule one says you must postpone to a Sunday, and rule two says to re-postpone until a Monday.

From Tuesday until Monday is six days. The full number of weeks in a common year (50) gives 7 x 50 = 350 days. So that year would have 356 days -- if rule three didn't intervene.

WHY RULE FOUR, THE MONDAY-COMMON YEAR FOLLOWING LEAP YEAR POSTPONEMENT?

The minimum number of days an intercalary year can have is 383 days. If rule four weren't in effect, some leap years would have only 382 days. An average leap year has 5 d 21 h 589 p over a full number of weeks, which is 54.

Suppose that a common year just after a leap year began on the same day as the molad, Monday 2 d 9 h 589 p. When would the preceding leap year have begun?

```
Monday                  2 d   9 h  589 p
average leap yr.      -(5 d  21 h  589 p) excess over full number of
                      ------------------- weeks.
                      - 3 d -12 h
```

-3 d -12 h is the same as -4 d +12 h. Whenever you are working with negative days of the week, you merely add 7 d to find out what day of the week you are actually on. -4 d +7 d is the 3 d of the week, or

Tuesday. The molad of the leap year thus occurred on a Tuesday at noon. But rule one says that this must be postponed to a Wednesday, and by rule two, re-postponed to a Thursday.

If the leap year began on a Thursday and ended on a Monday, there would be four days over the full number of weeks. 7 x 54 weeks is 378 days. 378 days plus 4 days is only 382 days; less than the minimum.

Without these postponement laws, the sacred calendar would be in a perpetual state of confusion. Holy Days would fall on a Sunday. The lengths of years would be irregular. Calendar reformers would be tempted to tamper with the sacred calendar more often. Picture the difficulty of a deacon trying to keep a Sabbath Day holy while frantically making last minute preparations for the Passover ceremony!

But all that turmoil is avoided by four very simple and easily applied postponement rules. Instead of the Sacred Festivals being subordinate to the Hebrew calendar the latter serves the Holy Days.

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PROGRAM VII

COUNTING THE DAYS OF THE WEEK AND THE DAYS OF THE MONTH

PERFORMANCE GOAL 7A:

You will count forward or backward a specified length of time from a given day of the week and correctly determine the new day of the week.

For example:

What day of the week is 164 days before Thursday?
Answer: Monday.

PERFORMANCE GOAL 7B:

Given the date and the day of the week for any one day in a Roman or Hebrew month, you will correctly indicate:

a) The day of the week for any other given date in that month, or
b) the dates for any specified day of the week in that month.

For example:

a) If the 13th of a month is on a Friday, what day of the week would the 4th be?
Answer: Wednesday.

b) What are the dates for Tuesday in a month when a Monday in that month is on the 9th?

Answer: 3, 10, 17, 24, (31).

COUNTING THE DAYS OF THE WEEKS 7A

Counting the days of the week is so simple that you're probably inclined to use "finger-calculations" quite often! Despite the apparent lack of complexity, however, mistakes can easily creep in. Wouldn't you prefer a less embarrassing technique than finger-counting to explain why a Holy Day occurs when it does? A more dignified demonstration is possible!

All of the Sacred Festivals of God are anchored to Tishri one. For example, the Day of Atonement is ten days after Trumpets. Passover is 164 days before. Accurate determination of the day of the week of the Festivals depends on correct counting procedures. Basic arithmetic is all that's needed.

As you recall from Program 3, Calculating the day of the week of the molad Tishri, multiples of full weeks do not affect the final day of the week involved. Only the excess, or remainder, is significant. Just as seven days from Monday is still a Monday, 147 days (twenty one weeks) before or after Monday is still a Monday.

In the same manner as Program 3, equate the days of the week with the numbers 1 to 7. Sunday is the first day, Wednesday is the fourth day, the Sabbath is the seventh day.

Days AFTER a reference day are ADDED to the numerical equivalent of the reference day. Days BEFORE the reference day are SUBTRACTED from the numerical equivalent. You will find it much easier if you take out full multiples of weeks before you add or subtract.

Here are some examples. On what day of the week will Atonement occur when Trumpets is on a Monday? Atonement (Tishri 10) is nine days after Trumpets, or one week and two days later. Monday is the second day of the week.

$2 + 2 = 4$ The fourth day of the week is Wednesday.

On what day of the week will Passover occur when Trumpets is on a Sabbath? Passover is 164 days before Trumpets. Divide 164 by 7, and you can express the time interval as 23 weeks and 3 days.

$7 - 3 = 4$ The fourth day is Wednesday. (It's observed Tuesday night.)

What happens when you want to find the day of Passover if Trumpets were on a Monday? Arithmetically, the problem is simply $2 - 3 = -1$. What day of the week does this negative number indicate? Simply ADD 7 to any negative number in these calculations of the day of the week:

$-1 + 7 = 6$ The sixth day of the week is Friday.

Here is an alternative way of expressing the same reasoning. First indicate the total time in terms of full weeks and days over a full week As an illustration, Passover is always 164 days before Trumpets. This is the same as 23 weeks and 3 days before. If Trumpets occurs on Tuesday, 23 weeks before the Tuesday of Trumpets is another Tuesday. What is three days before Tuesday? One day before is Monday; two days before is Sunday; three days before Tuesday is the Sabbath. Passover is on a Sabbath, observed Friday evening.

Pentecost always occurs on a Sunday because of inclusive reckoning. Leviticus 23:15 states that Pentecost is counted "from [beginning with] the morrow after the Sabbath, from the day that ye brought the sheaf of the wave offering ..." The wavesheaf offering was performed on Sunday, the day after the weekly Sabbath. With a Sunday the first day, seven full weeks (or forty-nine days) takes one to the fiftieth day, again on a Sunday.

By counting inclusively, 50 days is seven weeks. If you divide seven weeks by seven, you have no remainder. Sunday is the first day of the week, so arithmetically you have 1 + 0 = 1, or Sunday, as you started.

Practice finding the day of the week in the following problems. Set them up in terms of a simple arithmetical expression.

7a.1 What is 17 days after a Monday?

7a.2 What day is 6 days before Tuesday?

7a.3 What is 50 days after a Saturday.

7a.4 On what day does Passover occur if Trumpets is on a Tuesday?

7a.5 What day is 53 days before Thursday?

7a.6 If Passover is on a Friday (Thursday evening), when does Atonement occur?

The answers are on the next page.

7a.1 17 days = 2 weeks, 3 days. Monday is the second day of the week. 2 + 3 = 5; Thursday.

7a.2 Tuesday is the third day of the week. 3 - 6 = -3. Add 7 to change this to a real day: -3 + 7 = 4; Wednesday

7a.3 50 days = 7 weeks, one day. Sabbath is the seventh day. 7 + 1 = 8. Divide by seven and look at the remainder: 1; Sunday.

7a.4 164 days = 23 weeks, three days. Tuesday is the third day. 3 - 3 = 0. Convert this to a real day by adding seven: 0 + 7 = 7; Sabbath.

7a.5 53 days is 7 weeks, 4 days. Thursday is the fifth day. 5 - 4 = 1; Sunday.

7a.6 Passover and Trumpets are separated by 23 weeks, 3 days. Atonement is nine days after Trumpets, or one week, two days later. Passover is on the sixth day. Then Trumpets is on:

6 + 3 = 9. Eliminate the seven (9-7): 2; Monday.

When Trumpets is on Monday, Atonement will be:
2 + 2 = 4; Wednesday.

COUNTING THE DAYS OF THE MONTH, 7B

Hebrew months have either 29 or 30 days. Most months on a Roman calendar contain 30 or 31 days. As you recall from the form of most printed calendars, the same days of the week are placed over top one another.

For example, take a month of 30 days whose first day is Sunday. It will appear:

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

Take particular notice of the sequence in the columns (up and down the page). The Tuesday column contains all the days of the month which fall on Tuesday. They are, in this case, 3, 10, 17, 24.

What number separates these days of the month? Seven. One week from the third day of the month is 7 days later, or 3 + 7 days into the month. One week after the 10th is 10 + 7, or the 17th, etc. Learn to count by sevens for numbers up to 31.

Not every month will begin with the first day of the week, of course. You still can utilize the sequence of numbers separated by seven no matter what day of the week the month commences.

7b.1 What is the sequence of days of the month which occur on the Sabbath when the first Sabbath is on the 2nd?

7b.2 What is the sequence of days of the month that fall on Monday when the first day of the month is a Friday?

7b.3 If one of the Tuesdays of the month is the 22nd, what would be the other days of the month for Tuesdays?

7b.4 What are the days of the month for the Sabbath in a month where one of them is on the 16th?

7b.5 The month of AB has 30 days. The 10th of Ab is a Thursday. On what days of the month will the Mondays occur?

7b.6 The month of Thammuz has 29 days. The 6th of Ab is a Sabbath. On what days of the month will the Wednesdays occur?

7b.7 If the 14th of a month is on Wednesday, what day of the week would the 26th be?

7b.8 If the 22nd of the month is on Sunday, what day of the week would the 10th be?

The answers to these problems are below.

7b.1 2, 9, 16, 23, 30 are Sabbaths.

7b.2 Friday, the 6th day of the week, is the first of the month. Then Monday, three days after Friday, is the fourth. The Mondays are: 4, 11, 18, 25.

7b.3 The 29th is the last Tuesday in the month. To find the others, count backward "by sevens" from the 22nd: 15, 8, 1.

7b.4 16, 23, 30; 9, 2. These are the Sabbaths.

7b.5 10th = Thursday; Monday is 4 days later, or the 14th. 7, 14, 21, 28 are the Mondays.

7b.6 Sabbath = 6th of the month. Wednesday is three days earlier in the week, or the 3rd: The Wednesdays in Thammuz (in this case) are 3, 10, 17, 24.

7b.7 Two weeks after the 14th is the 28th, a Wednesday. The 26th is two days before the 28, so it is a Monday.

7b.8 Two weeks before the 22nd is the 8th, a Sunday. The 10th will be two days later, or Tuesday.

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PROGRAM VIII

DETERMINING THE DATES OF THE ANNUAL FESTIVALS

PERFORMANCE GOAL 8:

Given the day of the week and the Roman date for the Feast of Trumpets, you will correctly determine the day of the week and the day of the Roman month for each of the other annual Festivals in the Roman year. These Festivals are listed in Leviticus 23.

One of the most practical aspects of studying the Hebrew calendar is understanding the layout of Sacred Festivals. Obviously, if the Western World were using the Hebrew calendar for routine business affairs, there would be no need for transforming the dates of Trumpets, Atonement, Passover, etc., to the Roman calendar. But since this isn't the case, one must know what day of the Roman calendar each Holy Day corresponds.

Program 8 builds upon the calculation skills you have learned in program 7. Take time to review the previous program as is necessary for you.

In order to understand the relationship of the dates of the Holy Days, let's summarize the calendrical information given in Leviticus 23

Reference	Festival	Hebrew Date
Lev. 23:5	Passover	Nisan 14; 164 days before Tishri 1

:6-7	1st day of Unleavened Bread	Nisan 15
:8	7th day of Unleavened Bread	Nisan 21
:15-16	Pentecost	50 days beginning with the Sunday of the wavesheaf offering, (which is the day after the [a] regular weekly Sabbath) [NOT AN ANNUAL SABBATH] [THE SUNDAY WHICH IS] during the Days of Unleavened Bread. (Always a Sunday.) [See the WWN 5-11-87 & GN 6-74 for comments in brackets]
:24	Trumpets	Tishri 1
:27	Atonement	Tishri 10 (9 days after Trumpets)
:34-35	1st day of Tabernacles	Tishri 15
:36	Last Great Day	Tishri 22; (the "8th day" of the Festival, or 7 days after Tishri 15)

One of the first questions you will have is this: How do you know that Nisan 14, Passover, is 164 days before Tishri 1? The answer is quite simple: the intervening months are always the same length, no matter whether the year is leap or common. Nisan has 30 days; Zif (Iyar) 29; Sivan 30; Tammuz 29; Ab 30; Elul 29. Then comes Tishri. After the 14th day, Nisan has 16 days remaining in the month. The next five months, Iyar through Elul have 147 days (29 + 30 + 29 + 30 + 29). One more day to Tishri 1. Add 16 + 147 + 1, and you have 164 days.

Remember the "count by seven" pattern that separates the same day of the week in any month? You learned in Program 7 that days 1, 8,15, 22, and 29 of a month are the same day of the week. Notice that three of the four Holy Days in the fall season occur on the SAME DAY OF THE WEEK!

All the fall Holy Days occur in the month of Tishri, on the 1st, 10th, 15th, and 22nd. Once you determine the day of the week of Trumpets, you know immediately the day of the week of the first day of Tabernacles and the Last Great Day. They're all the same! And Atonement is simply two days later in the week than Trumpets.

Another convenient feature of the Holy Days is that Tishri 1 can fall only on a Monday, Tuesday, Thursday or Sabbath. (Recall Postponement Rule 2 in Program 6?) This means that in any Roman year, the Holy Days will have one of these four patterns based upon what day of the week the Feast of Trumpets occurs. You have a "Monday pattern", a "Tuesday pattern", a "Thursday pattern", and a "Sabbath pattern". Once you understand these four patterns -- and they need NOT be memorized, as you will see! -- you can list the day of the week of ALL the Sacred Festivals for any given year in about 30 seconds. Yes, it's that simple!

Here is a brief diagram to show you how all the Holy Days in a Roman year hinge upon the day of Tishri 1. The Hebrew civil year is reckoned from Tishri to Tishri, so in that sense, two Hebrew years are involved when you specify all the Holy Days in one Roman year.

```
           How the Annual Holydays Are Governed By Tishri 1

              HEBREW YEAR                HEBREW YEAR
  Tishri 1                  Tishri 1                Tishri 1
 (Trumpets)                (Trumpets)              (Trumpets)
      |                         |                        |
      |                         |                        |
--//--|                         |                        |
      |-------------------------|------------------------|
      |             |           |           |            |
      |---       Passover       |---     Passover        |---
      |   |                     |  |                     |  |
      | Atonement               |  Atonement             | Atonement
      | Tabernacles             |  Tabernacles           | Tabernacles
======================================================================
               |                        |                          |
           January                  January                    January
               |        ROMAN YEAR      |      ROMAN YEAR          |
               |                        |                          |
```

How do you lay the Holy Days out on the Roman Calendar? Since you determine Tishri l first, it's a good policy to work with the fall festivals first, then the Passover and Days of Unleavened Bread, and finally Pentecost. Of course, the day of the week for Pentecost is always on a Sunday because of the way God defined the time for observing that Holy Day. Nevertheless, you still need to find the date for Pentecost.

Take the "Sabbath pattern" as an illustration of the days of the week. Your reasoning will go something like this:

For this year, Trumpets (Tishri 1) is on a SABBATH
Atonement is 9 days later, or two days later in the week: MONDAY
Feast of Tabernacles begins on the 15th of Tishri: SABBATH
The Last Great Day is Tishri 22: SABBATH

(Or, in more abbreviated fashion, think of Tishri 1, 10, 15, 22 corresponding to Sabbath, Monday, Sabbath, Sabbath.)

Passover is 164 days before Trumpets, or three days earlier in the week: WEDNESDAY
The first day of Unleavened Bread is the next day: THURSDAY
The last day of Unleavened Bread is Nisan 21, one week after Passover: WEDNESDAY

(In a less verbose manner, think Nisan 14 is 3 days earlier in the week than Trumpets. Then Nisan 14, 15, 21 correspond to Wednesday, Thursday, Wednesday.)

Pentecost is a SUNDAY.

If you are still uncertain about how this pattern of days is thought out, please read Program 7 again. It will become much simpler once you understand the basic concepts of counting days that are explained in that program.

Now practice on the four patterns yourself. You might want to do them more than once just to impress the thinking process more fully in your mind. The four patterns of Holy Days are extremely important! Work each pattern independently from the others -- for your own benefit.

8.1 What is the "Thursday pattern" of Holy Days? In other words, what are the days of the week for each of the annual festivals when Trumpets is on a Thursday?

8.2 What is the "Monday pattern" of Holy Days?

8.3 What is the "Tuesday pattern" of Holy Days?

8.4 What is the "Sabbath pattern" of Holy Days?

The answers are given below. (Don't try to memorize them UNLESS you can first figure them out yourself!)

8.1 Tishri 1 is on Thursday. Tishri 1, 10, 15, 22 correspond to Thursday, Sabbath, Thursday, Thursday.
Passover is 3 days earlier in the week than Trumpets: Monday.
Nisan 14, 15, 21 correspond to Monday, Tuesday, Monday.
Pentecost is on Sunday.

On an examination, of course, you should actually list these dates along with the Holy Day:

Passover, (Nisan 14), Monday
1st day of Unleavened Bread, (Nisan 15), Tuesday
Last day of Unleavened Bread, (Nisan 21), Monday
Pentecost, Sunday Trumpets, (Tishri 1), Thursday
Atonement, (Tishri 10), Sabbath 1st day of Tabernacles, (Tishri 15), Thursday
Last Great Day, (Tishri 22),Thursday

8.2 Tishri 1 is on Monday. Tishri 1, 10, 15, 22 correspond to Monday, Wednesday, Monday, Monday.
Passover is three days earlier in the week: Friday.
Nisan 14, 15, 21 correspond to Friday, Sabbath, Friday.
Pentecost is on Sunday.

8.3 Tishri 1 is on Tuesday. Tishri 1, 10, 15, 22 correspond to Tuesday, Thursday, Tuesday, Tuesday.
Passover is three days earlier in the week than Trumpets: Sabbath.
Nisan 14, 15, 21 correspond to Sabbath, Sunday, Sabbath.
Pentecost is on Sunday.

8.4 See the example given previously in this program for the answer.

Now that you can easily determine the day of the week for each of the annual Holy Days, the final task is to specify the day of the month on a Roman calendar for each Festival.

The Roman dates of the fall festivals are very easy to determine if you simply work with the "seven pattern" of days in a month -- that you learned in Program 7. All you are concerned with are the equivalent Roman dates to Tishri 1, 10, 15, and 22.

You will either be given the day of the week and the day of the Roman month for Trumpets, or else you will calculate the molad Tishri for the required year yourself. The known correspondence between the Hebrew calendar and the Roman calendar for one day provides the key to determining the other dates.

Imagine a Hebrew calendar for the month of Tishri, but with most of the days except for 1, 10, 15, and 22 deleted.

```
                              TISHRI

                            1
                           (8)  (9)  10
                           15
                           22
```

Suppose that Tishri 1 in a particular year occurred on September 7. To find the dates of the other fall Holy Days, lay out an abbreviated calendar for the month of September (and October, if necessary):

```
     Tishri     1                September:   7
          (8)  (9)  10                     (14) (15) 16
          15                                21
          22                                28
```

Essentially, all you are doing in either calendar is counting by sevens from your starting day, the Feast of Trumpets. Atonement is nine days later in the month, so it might help you to think 1, 8-9-10, 15, 22 and correspondingly for this example, 7, 14-15-16, 21, 28.

As another illustration for the fall festivals, what happens when Trumpets is on September 25? Here you must remember to change months, but that's no horrendous problem!

```
     Tishri:      1      September :     25
             - - 10                     (32) (33) (34) = Oct. (2) (3)
            15                           39             = Oct.  9
            22                           46             = Oct. 16
```

September has 30 days, so you can simply call October 1, September "31" for calculation purposes. September 46 is simply 16 days after September 30, or October 16.

What happens when Trumpets is on August 28? The other fall Festivals will occur in the month of September. In order not to get involved with negative numbers, go ahead and add seven days, which is equivalent to finding Tishri 8. What day is August 35? September 4, because August has 31 days. September 1 is the same as August 32; September 2 the same as August 33, etc.

Your abbreviated calendar will appear:

```
     Tishri     1                       August    28
               (8)  (9)  10             Sept.     (4)  (5)  6
               15                                 11
               22                                 18
```

Observe that you are making a direct correspondence between the Hebrew calendar and the Roman calendar. Counting by full weeks -- "by sevens" -- enables you to avoid other computational confusions that can creep into your work if you just add days. You can add 9 days to the Roman date of Tishri 1 and find the date for Atonement; 14 days to Trumpets for the first day of Tabernacles, and 21 days for the Last Great Day. Counting by sevens with an abbreviated calendar is less error-prone, once you understand the principle.

Here are a few problems for you to practice determining the Roman dates for the annual Festivals. Just indicate the dates for Atonement, the first day of Tabernacles, and the Last Great Day, but include the day of the week.

8.5 Trumpets is September 5, Monday
8.6 Trumpets is September 12, Thursday
8.7 Trumpets is September 23, Tuesday
8.8 Trumpets is August 25, Tuesday
8.9 Trumpets is September 16, Thursday
8.10 Trumpets is August 26, Sabbath

The problems are worked for you on the next page.

```
8.5  September  '5'        Monday     (Feast of Trumpets)
                 12 - '14'  Wednesday  Atonement, September 14
                '19'        Monday     Tabernacles, Sept, 19 (first day)
                '26'        Monday     Last Great Day, Sept. 26
```

The single quotation mark dates are the answers, following the same format as above:

```
8.6  September '12'           Thursday       (Trumpets)
                19 - '21'     Sabbath        Atonement
               '26'           Thursday       First day of Tabernacles
               '33' (Oct. 3)  Thursday       Last Great Day


8.7  September '23'                          Tuesday
                30 - '2'      October 2      Thursday
                '7'           October 7      Tuesday
               '14'           October 14     Tuesday


8.8  August '25'                                  Tuesday
          32  - '34', or September  1  -  '3'  Thursday
                                   '8'           Tuesday
                                  '15'           Tuesday

8.9  September '16'                          Thursday
                23 - '25'                    Sabbath
               '30'                          Thursday
               '37' which is October '7'     Thursday
```

```
8.10 August   '26'                          Sabbath
               33 - 34, or September 2 - '4' Monday
                                    '9'      Sabbath
                                   '16'      Sabbath
```

Take a look at the spring season. Once you find the date for Passover (which you'll learn how to do very shortly), all you need to do is find the equivalent dates for Nisan 15 and Nisan 21. An abbreviated calendar for Nisan looks like this:

```
               Nisan     14   15
                         21
```

```
     If Passover were April 18, you would have:  April 18   19
                                                       25
```

The procedure is the same as for the fall season. Just remember that March has 31 days, so March 32 is the same as April 1. (You handled the 31 days in August the same way.)

Now indicate the Roman dates and day of the week for the first day of Unleavened Bread and the last day of Unleavened Bread in the following problems:

8.11 Passover is April 10, Monday
8.12 Passover is March 25, Friday
8.13 Passover is March 29, Friday
8.14 Passover is March 23, Wednesday

The problems are worked for yo on the next page.

```
8.11 Passover is April  10  '11'   Monday    Tuesday
                       '17'        Monday
```

To be sure you understand the pattern of this answer,
Passover is Monday, April 10
1st day of Unleavened Bread is Tuesday, April 11
7th day of Unleavened Bread is Monday, April 17

```
8.12 Passover is March 25  '26'                   Friday    Sabbath
                           '32' which is APRIL 1  Friday
```

```
8.13 Passover is March 29  '30'                   Friday    Sabbath
                      '36'      which is APRIL 5  Friday
```

```
8.14 Passover is March 23  '24'              Wednesday Thursday
                      '30'                   Wednesday
```

Now turn your attention to finding the date of Passover, once you know the Roman date for Trumpets. As you learned earlier in this program, Passover is 164 days before the Feast of Trumpets. What you do is count by whole Roman months until you obtain a number slightly larger than 164. (You can also find a number of days below 164.)

An example will clarify the procedure. Suppose Trumpets is on September 13. You want a number bigger than 164 days, so you mentally keep track of the months you are adding:

September:	13 days
August:	31 days
July:	31 days
June:	30 days
May:	31 days
April:	30 days
	166 days

166 days brings you to the last day of March. How many days past 164 did you go? 166 - 164 = 2. Two days into April is April 2 -- Passover.

Doing this the other way, had you only counted as far as May, you would have come up with 136 days. 136 days brings you to April 30. How many days into April must you go? 164 - 136 = 28 days. April 30 minus 28 days is April 2 -- Passover.

Take another illustration, Trumpets being on August 30. You have:

August:	30 days
July:	31 days
June:	30 days
May:	31 days
April:	30 days
March:	31 days
	183 days

183 days takes you to the last day of February, How many days into March is Passover? 183 - 164 = 19 days. Passover is March 19.

Once you practice on a few problems, you will have no difficulty in determining the Roman date for Passover. In these problems, also indicate the day of the week:

8.15 Trumpets is September 8, Thursday
8.16 Trumpets is September 11, Sabbath
8.17 Trumpets is September 5, Monday
8.18 Trumpets is August 25, Sabbath

The answers are listed for you below.

8.15 Passover is March 28, Monday
8.16 Passover is March 31, Wednesday
8.17 Passover is March 25, Friday
8.18 Passover is March 14, Wednesday

You've now learned how to find the day of the week and the Roman day of the month for all the Holy Days except Pentecost. Of course, you already know the day of the week. But how about the Roman date?

Before you can count fifty days to Pentecost, you need the starting point, which is the Sunday of the wavesheaf offering. The table below will give you an overview of the relationship of Passover and the wavesheaf offering:

Passover (Nisan 14)	Wavesheaf offering	Days after Passover
Friday (Thurs. eve)	Nisan 16 (Sunday)	2
Wednesday	Nisan 18	4
Monday	Nisan 20	6
Sabbath	Nisan 15	1

There's really no need to memorize this table, because you can easily determine the number of days from the Passover to the wavesheaf offering once you find the day of the week Passover occurs. Just count from the Passover to the Sunday of the wavesheaf offering.

When the Passover is on Monday, March 28, the Sunday of the wavesheaf offering is 6 days later, March 34 = April 3. With April 3 as the first day (counting inclusively), Pentecost, the fiftieth day, will be 49 days later. There are 30 - 3 days left in April, or 27 days. Pentecost is 49 - 27 days into May, or on May 22. As another way for determining the date of Pentecost, count seven full weeks from April 3. Here you are counting by sevens: April 10, 17, 24, May 1, 8, 15, 22. Again you have arrived at May 22 for the date of Pentecost.

Here's another example for determining the date of Pentecost. If Passover is on April 8, a Wednesday, the wavesheaf offering will be April 12, Sunday. Count seven full weeks from April 12: 19, 26, May 3, 10, 17, 24, 31. Pentecost is May 31.

Work out a few problems in order to firm up the process in your mind. Determine Pentecost for each of the following cases

8.19 Passover is Friday, March 30
8.20 Passover is Wednesday, March 16
8.21 Passover is Sabbath, April 2
8.22 Passover is Monday, April 20

The answers are on the next page.

8.19 Pentecost is Sunday, May 20

8.20 Pentecost is Sunday, May 8

8.21 Pentecost is Sunday, May 22

8.22 Pentecost is Sunday, June 14

You have learned how to work with the Holy Days in separate steps, fall festivals, Passover and Pentecost. Now integrate those skills and find for all the Holy Days in a Roman year the respective date and day of the week.

8.23 Trumpets is Monday, September 16

8.24 Trumpets is Sabbath, October 3

8.25 As the last problem in this series of programs, determine the day of the week and the Roman date for all the annual festivals in the year 2055 AD. You may use the chart included in Program 4.

The answers are below.

8.23	Passover	Friday, April 5
	1st day of Unleavened Bread	Sabbath, April 6
	7th day of Unleavened Bread	Friday, April 12
	Pentecost	Sunday, May 26
	(Trumpets	Monday, September 16)
	Atonement	Wednesday, September 25
	1st day of Tabernacles	Monday, September 30
	Last Great Day	Monday, October 7
8.24	Passover	Wednesday, April 22
	1st day of Unleavened Bread	Thursday, April 23
	7th day of Unleavened Bread	Wednesday, April 29
	Pentecost	Sunday, June 14
	(Trumpets	Sabbath, October 3)
	Atonement	Monday, October 12
	1st day of Tabernacles	Sabbath, October 17
	Last Great Day	Sabbath, October 24

8.25 See Program 5, pages 44 and 45 for the determination of Tishri 1

Passover	Monday, April 12
1st day of Unleavened Bread	Tuesday, April 13
7th day of Unleavened Bread	Monday, April 19
Pentecost	Sunday, June 6
Trumpets	Thursday, September 23
Atonement	Sabbath, October 2
1st day of Tabernacles	Thursday, October 7
Last Great Day	Thursday, October 14

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SAMPLE TEST QUESTIONS

1. How many lunar months are in a 19-year cycle?

2. How many 19-year cycles, leap years, and common years are between the bench mark and 171 AD?

3. Without using your chart, determine the REDUCED number of days, hours, and parts in TWO common Hebrew years.

4. Demonstrate (or derive) without charts the amount that the Hebrew common year leads or trails the Roman calendar.

5. What corrections (Roman leap year and Julian / Gregorian) are applicable for the following years:

971 BC   1716 AD   1291 AD   1900 AD   2155 AD

6. In 1984 AD the molad Tishri is September 25, Tuesday 11 h 976 p. What is the day of the week and the day of the month for Trumpets?

7. If the 29th of Elul is a Wednesday, what day of the week is the 15th of Elul?

8. What day of the week is 313 days before a Friday?

9. In 1981 AD Trumpets is on September 28, Monday. List the day of the week and the day of the Roman month for each of the annual festivals given in Leviticus 23.

10. List the day of the week and the day of the Roman month for all the annual festivals in the year 29 AD. You may use your chart.

HEBREW CALENDAR

Name of Month*	#Sacred	#Civil	Begins with new moon of
Aviv, or Nisan	1st	7th	March-April
Ziw	2nd	8th	April-May
Siwan	3rd	9th	May-June
Tammuz	4th	10th	June-July
Av	5th	11th	July-August
Elul	6th	12th	August-September
Tishri, or Ethanim	7th	1st	September-October
Marcheshwan	8th	2nd	October-November
Kislew	9th	3rd	November -December
Teveth	10th	4th	December-January
Shevat	11th	5th	January-February
Adar	12th	6th	February-March

(*) Modern Hebrew transliteration. Please compare with that below:

The spelling given in W. M. Feldman's, "Rabbinical Mathematics and Astronomy," is as follows:

Nisan	Tishri
Iyar	Marcheshvan
Sivan	Kislev

Tammuz	Tebeth
Ab	Sh'bat
Elul	Adar
	V'Adar

Use this as a guide for pronunciation.

THE SIX TYPES OF HEBREW YEARS

	COMMON			LEAP		
Month	Deficient	Regular	Perfect	Deficient	Regular	Perfect
Tishri	30	30	30	30	30	30
Marcheshwan (Heshwan)	29	29	30	29	29	30
Kislew	29	30	30	29	30	30
Teveth	29	29	29	29	29	29
Shevat	30	30	30	30	30	30
Adar	29	29	29	30	30	30
V'Adar	__	__	__	29	29	29
Nisan (Aviv)	30	30	30	30	30	30
Ziw (Iyar)	29	29	29	29	29	29
Siwan	30	30	30	30	30	30
Tammuz	29	29	29	29	29	29
Av	30	30	30	30	30	30
Elul	29	29	29	29	29	29
	353	354	355	383	384	385

(*) Modern Hebrew transliteration. See the top of this page for alternate spelling and pronunciation.